Numerical & analytic treatment of supernova feedback:

The right rule for the wrong reasons?

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Classical Arguments:

Energy conservation arguments first forwarded quantitatively by Larson (1974) equate the *gravitational potential gained* by the gas ejected from a galaxy with the *energy available from Suprnovae*.

$$M_{\rm out} v_c^2 \approx \epsilon_{\rm SN} M_\star$$

(Where $\epsilon_{\rm SN}$ is the mean energy converted per mass of stars formed.)

Together with the constraint on the total mass in the halo:

$$M_\star + M_{\rm out} \approx f_{\rm b} M_{\rm v}$$

this gives:

$$M_\star \left(1 + \frac{v_c^2}{\epsilon_{\rm SN}}\right) \approx f_{\rm b} M_{\rm v}$$

or

$$M_\star \approx \frac{f_{\rm b} M_{\rm v}}{1 + \epsilon_{\rm SN}/v_c^2}$$

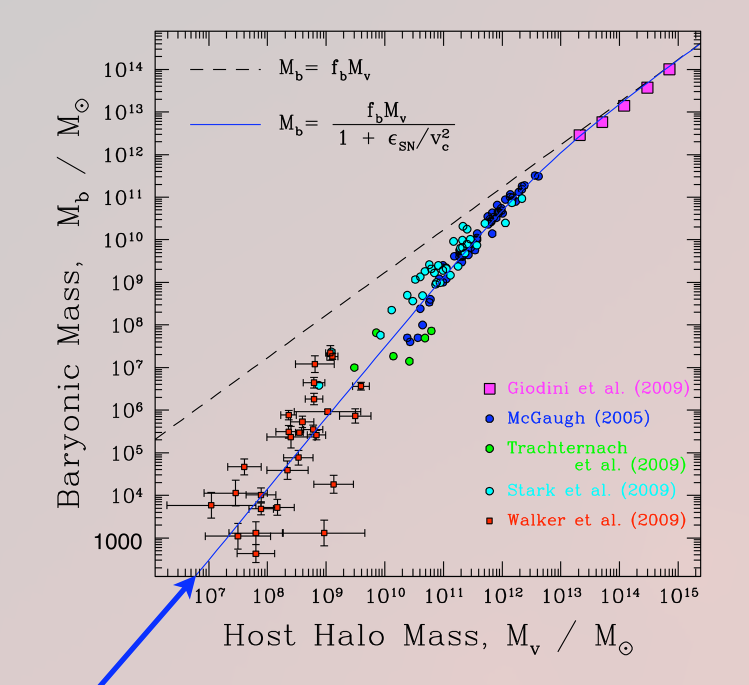


Figure 1. Galactic baryonic mass estimates from several publications, shown as a function of the virialised host halo mass (estimated from observed circular velocity or velocity dispersion). The lines show the simple prediction of self-similarity (dashed line) and the prediction based on energy conservation between supernovae and gas outflow, with the value $\epsilon_{\rm SN} = (300{\rm kms}^{-1})^2$ chosen for illustrative purposes only (solid line). This plot is deliberately similar in layout and content to figure 1 of McGaugh et al. (2010) and uses the same conversion from host halo mass to characteristic speed: $M_{\rm v}/10^{12}{\rm M}_\odot = (v_{\rm c}/187{\rm kms}^{-1})^3$.

Too good to be true?

(And why should the conversion of supernova energy be the same for all systems anyway?)

What can simulations tell us?

To revisit these traditional arguments, we have re-run existing SPH simulations of idealised disk galaxies at a range of particle mass. With the same kinetic supernova feedback, the outflow mass varies greatly with resolution. We try to link this with the mean gas outflow velocity, $\bar{v}$, and the characteristic potential, $\Delta\Phi$

When $\bar{v}^2 >> \Delta\Phi$, the results simply return approximate conservation of supernova wind momentum:

$$M_{\rm out} \approx \frac{p_{\rm SN}}{\bar{v}} \approx \frac{\frac{1}{2} M_w v_w}{\bar{v}}$$

At high resolution this condition is not met. The approximation is too simplistic here!

The effects of supernovae are simulated by assigning particles surrounding the given site a velocity kick in a random direction. Particles are chosen at random such that their total mass is twice the initial mass of stars formed.

For the massive galaxy, the outflow fraction at low resolution can be thought of as the fraction of these wind particles whose velocity kicks are orientated so that they can escape the disks gravity.

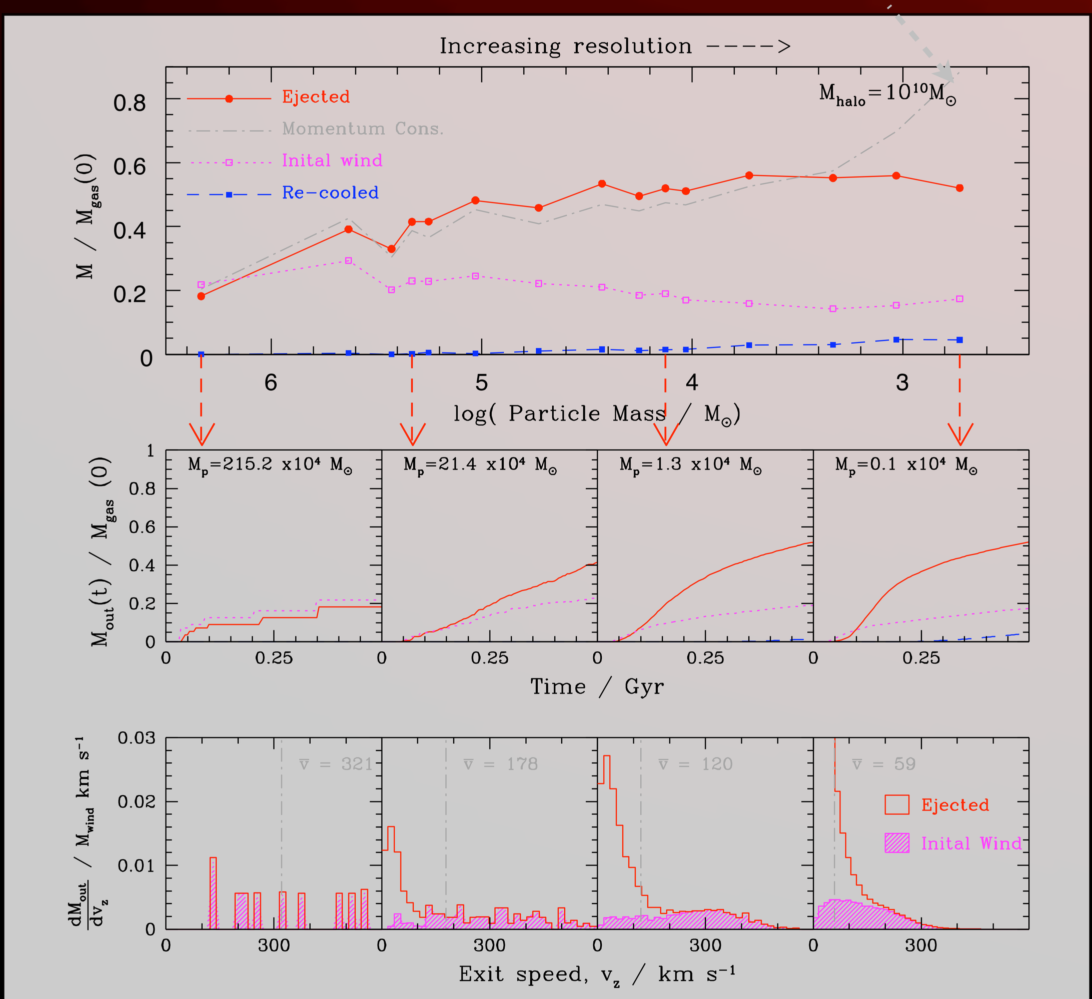


Figure 2. The effect of resolution on the mass outflow and evolution of an idealised, isolated dwarf galaxy (Halo Mass= $10^{10}{\rm M}_\odot$, Baryonic Fraction=0.054). The *same* initial conditions have been evolved using the GADGET code but with 15 different choices of particle mass. The softening length in all cases is 10pc. The **middle panels** show the time evolution of four of these, showing the gas mass fraction which has escaped (solid line), and the fraction which has escaped from the disk but returned (dashed line). Also shown is the fraction which have become "wind particles (dotted line), which in this set of simulations is just $M_{\rm w} = 2\dot{M}_\star$. The **upper panel** shows the final value of each of these quantities as a function of the particle mass used in the simulation run. Added to this panel is a faint dot-dashed line showing the result of straightforward conservation of initial momentum out of the plane (5). The lower continuation of the line at high resolution illustrates the importance of gravitational influence at low exit speeds (6). The **lower panel** shows the distribution of escaped gas as a function of the outward velocity component, v_z, at the point when it crosses the nominal galaxy "boundary" (see main text). The vertical dot-dashed line shows the mass-weighted mean, $\overline{v_z}$ (in ${\rm kms}^{-1}$) which is used in (5). Note that the integral under the total ejected mass is equal to $M_{\rm out}/M_{\rm wind}$, but the shaded area (including only wind particles) is not equal to unity because some wind particles do not escape.

Dwarf Galaxy

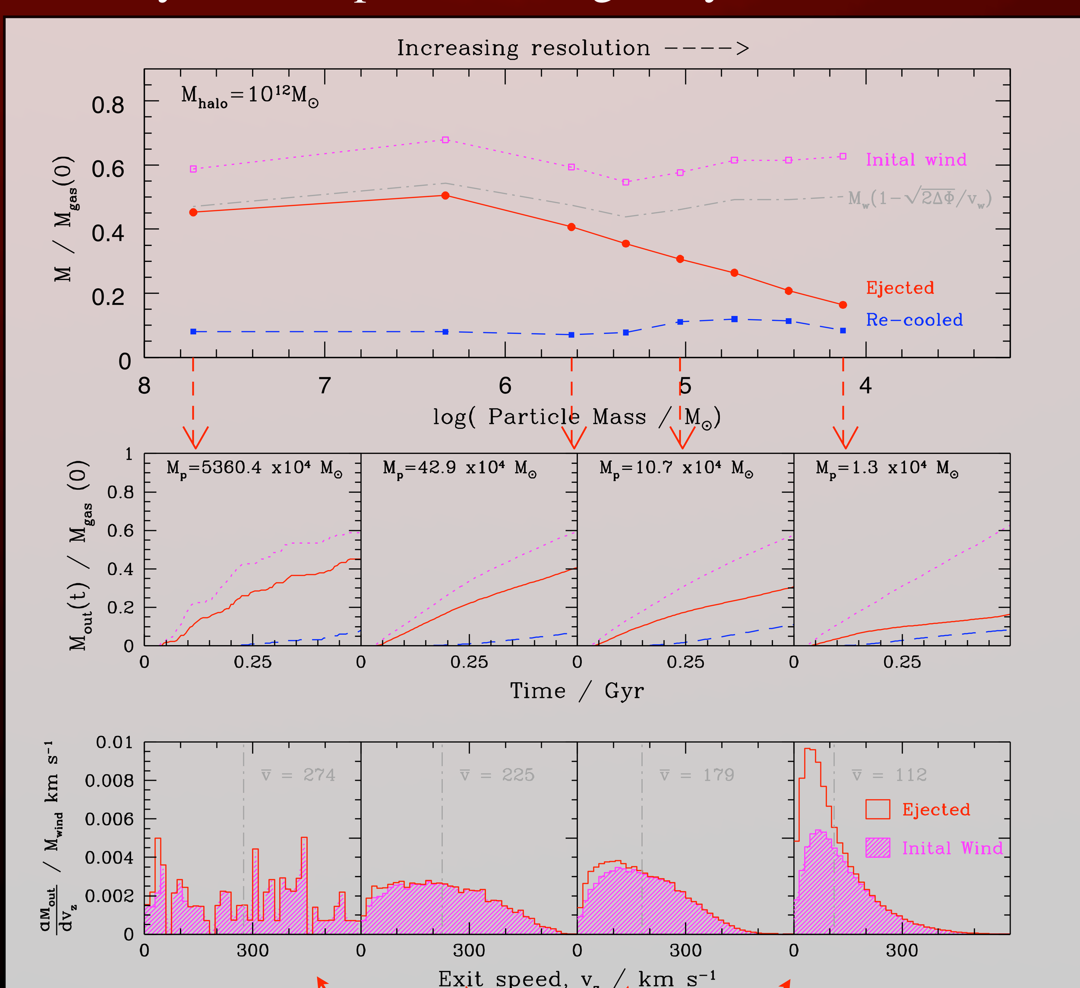


Figure 3. The effect of resolution on the mass outflow and evolution of an idealised, isolated Milky Way-like galaxy (Halo Mass= $10^{12}{\rm M}_\odot$, Baryonic Fraction=0.054). The key is the same as Fig. 2. The faint dot-dashed lines for the limiting behaviour are different, as explained in §2.2.2.

The outflow velocity gets lower and lower as the galaxy is simulated with more and more particles.

Massive disk galaxy

Mass-dependence of SN feedback

Using the approach outlined in Fig. 4, we can consider what would happen if we were able to simulate all systems at the resolution we are able to achieve with the dwarf.

At high resolution, the outflow is mass-dependent.

At low resolution, we more or less just recover the sub grid model, in this case:

$$\Delta M_{\rm out} \approx 2\Delta M_\star$$

Star formation is noatbly robust to changes in resolution

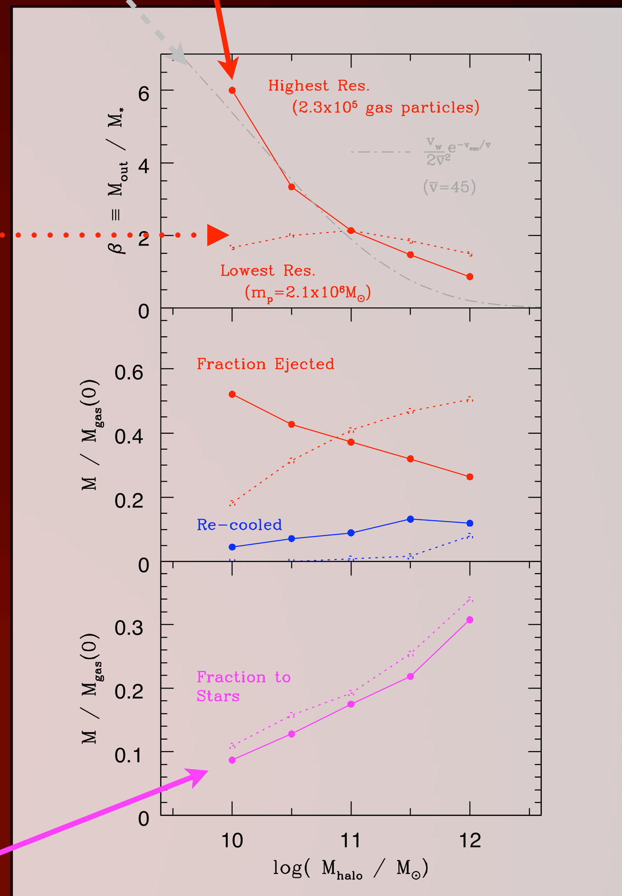


Figure 5. The outflow of gas from idealised galaxies of different total mass, $M_{\rm halo}$. The **lower panel** shows the fraction of the initial gas mass that has formed stars after being evolved for 0.5Gyr (circles), and the **middle panel** shows the fractions which have been ejected from the system and re-cooled. The **top panel** plots the ratio between the ejected mass and the mass of stars formed. In all panels the highest resolution is shown as solid lines/points and the lowest choice as dashed lines and open points. Superimposed on the top panel is a prediction based on a common velocity distribution (11) for the outflowing gas in the different galaxies, truncated according to the different potential barrier presented to escaping gas, which scales $\Delta\Phi \propto M^{2/3}$.

The high-resolution behaviour of these isolated simulations implies that there is some *initial velocity distribution:* set by momentum conservation...

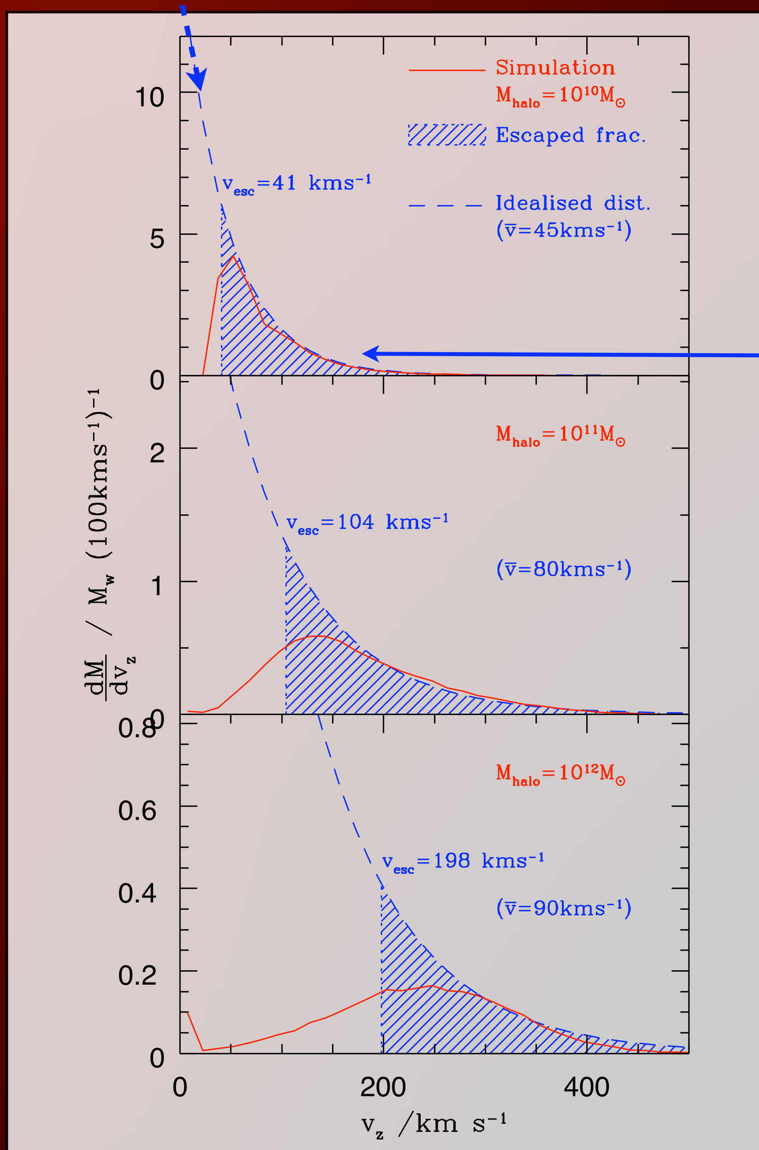


Figure 4. A illustration of the modelling approach described by equation (10) by comparison with the the results of simulations of three disk galaxies (total halo masses as labelled) run with the same number of particles ($\approx 2 \times 10^5$ gas particles). The solid line is the distribution of ejected gas as a function of estimated outward velocity immediately after the respective supernovae events. Superimposed as a dashed line is a simple velocity distribution (12) with mean, $\bar{v}$ chosen to closely match the tail of the distribution from each particular simulation. The cut-off velocity, v_{esc} is the value that separates the appropriate escape fraction for each initial distribution.

Example distribution

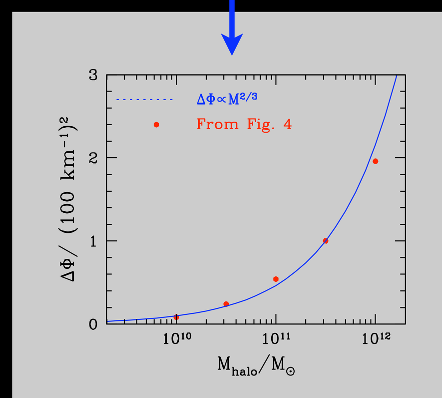
$$\frac{{\rm d}M}{{\rm d}v} \approx \frac{p_{\rm SN}}{\bar{v}^2} e^{-\frac{v}{\bar{v}}}$$

...of which only the high-velocity tail eventually escapes.

$$M_{\rm out} \approx \int_{v_{\rm esc}}^{\infty} \frac{{\rm d}M}{{\rm d}v} {\rm d}v$$

$$\approx \frac{p_{\rm SN}}{\bar{v}} e^{-v \quad /\bar{v}}$$

where the approximate cut-off in outflow velocity is set by the effective potential barrier to escape from the disk, so scales as: $v_{\rm esc}^2 \propto M^{2/3}$.



If we investigate the outflow out of the plane of the disk (as is plotted here) then we can write this in terms of the velocity distribution and the velocity required, on average, to escape:

$$M_{\rm out} \approx \int_{v_{\rm esc}}^{\infty} \frac{{\rm d}M}{{\rm d}v_z} {\rm d}v_z$$

which, for the random orientations at low resolution, is roughly:

$$\approx M_{\rm w} \left(1 - \frac{v_{\rm esc}}{v_{\rm w}}\right) . \qquad [\, v_{\rm esc} \sim \sqrt{2\Delta\Phi}\,]$$

As the resolution is increased, this limiting case is no longer relevant. The wind particles are interacting with the reast of the gas in the simulation and forming a quite different velocity distribution. But we could still model the outflow in this way, if only we knew what that velocity distribution should be...

GADGET simulations by Claudio Dalla Vecchia & Joop Schaye, after Springel, Di Matteo & Hernquist