Implementation of feedback in SPH: towards concordance of methods

Fabrice Durier & Claudio Dalla Vecchia

Abstract
We perform simulations of feedback from supernovae with smoothed particle hydrodynamics (SPH). We show for the first time that, in the absence of radiative cooling, concordance of thermal and kinetic feedback is achieved when using an appropriate time integration. In order to preserve a high level of energy conservation when using the hierarchical time-step scheme, we implemented in the Gadget-2 code a modified version of the time-step limiter proposed by Saïtoh & Makino (2009). We apply the limiter to general test cases, and find necessary, not only to ensure a fast information propagation, but also to enforce a prompt response of the system to the energy perturbation. The method proposed here to handle strong feedback events enables us to achieve energy conservation at percent level in all tests, even if all the available energy is injected into only one particle. Finally, we show that, even if cooling processes are taken into account and providing a sufficiently high resolution, simulations of an individual supernovae explosion with the different feedback methods are still reaching concordance.

Time Integration Schemes

Energy is given at time $t_i$ to particle $i$, and particle $i$ is one of its neighbours. (labels given on the right side of each sketch shows the level of the time-step hierarchy).

- **Individual**: particles are only able to adapt their time-step when they become active. Therefore, heated/kicked particles may complete a significant number of steps before their neighbours become active as well.
- **Limiter**: neighbouring particles communicate to each other the length of their time-steps, and keep the ratio of long over short steps no larger than a fiducial factor of $4$. However, particles still need to be active before their neighbours adjust their time-step accordingly to the limiter criterion.
- **Limiter + Update**: here we make sure that heated/kicked particles become active at the time of energy injection, and adjust their time-step accordingly to the amount of energy they receive. When applied in combination with time-step limiter, impacted particles and their neighbours can promptly react to the change of energy in the medium.

Halo's test

We present below the results of an off-centre explosion in a self-gravitating gas sphere using the same integration schemes as before. Both thermal and kinetic energy injection methods are considered.

- **Individual (top-left)**: the behaviour is similarly wrong in both cases. The halo atmosphere is disrupted.
- **Limiter only (top-right)**: an adaptive time-step is applied in combination with the time-step limiter. Particles are still able to travel to very large distances before interacting with the medium.
- **Limiter + Update (bottom-left)**: neighbouring particles communicate to each other the length of their time-steps before their neighbours become active as well. Therefore, heated/kicked particles may complete a significant number of steps before their neighbours become active as well.
- **Limiter + Update (bottom-right)**: here we make sure that heated/kicked particles become active at the time of energy injection, and adjust their time-step accordingly to the amount of energy they receive. When applied in combination with time-step limiter, impacted particles and their neighbours can promptly react to the change of energy in the medium.

What about Cooling?

Simulations of a single SN explosion including (orange lines) or not (black lines) cooling processes are compared in the figure below for both feedback methods. From top to bottom we see the evolution, of the thermal (solid) and kinetic (dashed) budget, of the blast radius and of both the bubble (dash) and shell (solid) temperatures. Red lines give the expected Sedov radius and shell temperature as a reference.

Firstly, we see that simulations need a certain amount of time to numerically converge to the Sedov phase (the expected energy partition is given by the grey horizontal dotted lines). This convergence time can be expressed as a function of the physical properties of the problem and of the numerical resolution as follow:

$$t_{\text{conv}} = 1.4 \times 10^{51} \frac{E_{\text{tot}}^{1/3} M_{\odot}^{1/3}}{n_0^{1/3}} \approx 140 \text{ yrs}$$

Secondly, following Cox-1972 and Blondin et al.-1998 we define the transition from the Sedov to the snowplough phases by the time at which the cooling time of the shell equals the blast age. For our choice of the cooling function, this transition time can be expressed by:

$$t_{\text{conv}} = 3.2 \times 10^{52} \frac{E_{\text{tot}}^{2/3} \rho_0^{1/3}}{n_0^{1/3}} \approx 10^7 \text{ yrs}$$

Finally, we see from the cooling runs that both simulations are able to describe correctly the snowplough phase since they both reproduce the extension of the snowplough radius as defined by McKee & Ostriker-1977 (given by the green lines).

Computational time [%]

<table>
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<tr>
<th>Integration scheme</th>
<th>Energy conservation [%]</th>
<th>Computational time [mm]</th>
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<tbody>
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<td>Limiter + Update</td>
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