Investigating Optimal AGN feedback

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Introduction

AGN feedback in massive galaxies and galaxy clusters can be thought of as a naturally occurring control system which plays a significant role in regulating both star formation and the X-ray luminosity of the surrounding hot gas. By analogy with the *principle of least action* in classical mechanics, AGN feedback must also be optimal, in some sense. Therefore, the observed behavior of a system, if interpreted correctly, should provide valuable information about the underlying physical principles which govern feedback. For example, AGN heating in elliptical galaxies and galaxy clusters seems to occur in relatively discrete episodes [1][2], as opposed to operating continuously at an approximately constant rate. Empirically, this occurs because AGN and star formation in Brightest Cluster Galaxies (BCGs) seem to be triggered whenever the central cooling time of the Intra-cluster Medium (ICM) falls below 0.5 Gyr [3][4]. However, if this is the only consideration, why isn't AGN heating perpetually triggered and quenched on very short timescales as the cooling time varies around the triggering value? Indeed, if this was the case, the AGN radio emission would appear to be continuous, in conflict with observations. One possibility is that there are time delays associated with the inflow of material on to the black hole and the dissipation of energy output from the AGN. Such time-delayed AGN heating would cause the system to overshoot its equilibrium state and oscillate between longer-lived "on" and "off" phases. Here we take a different approach and employ optimal control theory as a method with which to explore AGN feedback. Intriguingly, it is shown that AGN heating which minimizes the total energy output of the system (gas cooling + AGN heating) implicitly balances gas cooling while also minimizing the black hole growth rate. Furthermore, such optimal heating necessarily occurs in discrete and periodic events with a duty cycle that is governed by the feedback strength. A direct consequence of this effect is that heating events will be sufficiently powerful to expel hot gas from the gravitational potential of a galaxy, but not a galaxy cluster. This is consistent with theoretical explanations for the observed steepening of the L_X-T relation for temperatures below 1-2 keV [5][6].

Methods

Assume that the temporal evolution of the X-ray gas luminosity, L_X, due to its own radiative cooling and applied AGN heating at a rate, H, can be approximated by an Nth-order linear differential equation of the form:

$$a_N \frac{d^N L_X}{dt^N} + a_{N-1}\frac{d^{N-1} L_X}{dt^{N-1}} + ... + a_0 L_X = H$$

where a_N, a_{N-1} and a_0 are constant coefficients.

Observations of galaxy clusters [7][8] indicate that AGN heating rates scale with the X-ray luminosity of the ICM, but heating is only triggered if the central cooling time falls below a critical threshold [3][4]. Therefore, AGN heating is expressed as $H = \alpha(t)kL_X$, where k (>1) is the feedback strength and $\alpha(t)$ varies in the range 0-1 to represent the observed AGN triggering criterion.

To predict how $\alpha(t)$ might vary, using the minimum number of assumptions, let us investigate the possibility that linear feedback heating acts to minimize the total energy output from the system. This constraint ensures that heating balances cooling with the minimum black hole growth rate. Then, for N =1, dimensional analysis suggests a_0 =1 and a_1 = -τ, and the optimal control problem for AGN feedback can be mathematically expressed as [9][10]:

$$\text{maximize } -E_{out} = -\int_{t_0}^{t_1}[L_X + H]dt$$

$$\text{subject to } \frac{dL_X}{dt} = \frac{L_X - H}{\tau}$$

$$\text{and } L_X(t_0) = L_{X0}$$

where minimizing E_{out} is equivalent to maximizing $-E_{out}$.

For generality, the end state, $L_X(t_1)$, is left unspecified. In this case, the set of necessary boundary conditions is completed by $\lambda(t_1)$=0, where $\lambda(t)$ is the so-called costate variable. This time-varying quantity adjoins the minimization constraint to the system dynamics in the optimal control Hamiltonian. According to Pontryagin's maximum principle [9][10], the optimal control maximizes the system Hamiltonian (F) which, for the case above, is given by:

$$F = -[L_X + H] + \lambda\left[\frac{dL_X}{dt} - \frac{(L_X - H)}{\tau}\right]$$

Since F is linear in α, it is minimized when α takes its largest possible value (if $\partial F/\partial\alpha > 0$) or its smallest possible value (if $\partial F/\partial\alpha < 0$). The partial derivative of the Hamiltonian with respect to the control variable is:

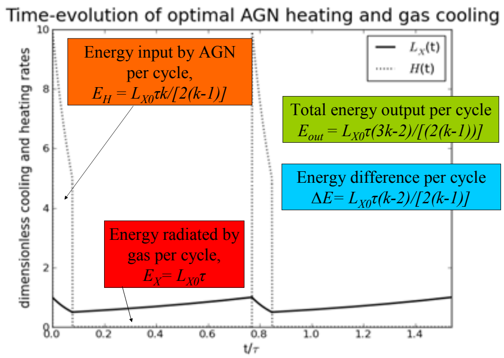
$$\frac{\partial F}{\partial \alpha} = kL_X\left[\frac{\lambda}{\tau} - 1\right]$$

Thus, α will be 0 as $\lambda(t)\rightarrow 0$ towards the end of a heating/cooling cycle. However, earlier in the cycle, $\lambda(t)$ may be large enough to make $\partial F/\partial\alpha$ positive, in which case α will be 1. Solutions such as this, which require the control variable to be held at one extreme and then shifted to the other are called "bang-bang" controls [9][10].

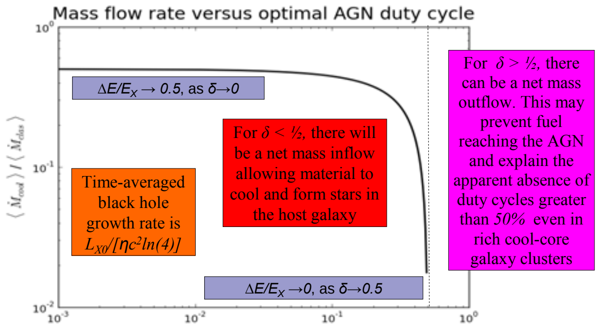
The optimal switching time, t_*, which minimizes E_{out} is found using: $\frac{\partial E_{out}}{\partial t_*} = 0$

Results

Only if the heating/cooling cycle begins/ends when $L_X(t_1) = L_X(t_0)$ does $L_X(t)$ evolve periodically. In this case, the duration of a single heating/cooling cycle is $t_1 - t_0 = \tau \ln(2)\, k/(k-1)$. Furthermore, the duty cycle of AGN heating is $\delta \equiv t_*/(t_1 - t_0) = 1/k$. The figure below shows the periodic evolution of $L_X(t)$ subject to optimal heating, with a feedback strength of $k=10$ so that $\delta=1/10$.



By definition, $\Delta E/E_X \equiv$ *actual mass flow rate / classical mass flow rate* $= (1-2\delta)/[2(1-\delta)]$, and is shown below.



Conclusions

By applying optimal control theory to AGN feedback, we arrive at the following conclusions:

- The total energy output of the system (gas cooling + AGN heating) is minimized if AGN feedback supplies heat in the form of discrete outbursts. This scenario minimizes the black hole growth rate while still ensuring that AGN heating balances gas cooling.
- If the heating/cooling cycle begins/ends when $L_X(t_1) = L_X(t_0)$, the solution of $L_X(t)$ will be periodic. In this case, there will necessarily be a difference between the time-averaged AGN heating and gas cooling rates, permitting fuel to reach the AGN and on-going star formation in the host galaxy.
- The optimal AGN heating duty cycle, δ, for a first-order system with linear feedback ($H = kL_X$) is shown to be the inverse of the feedback strength, k, such that $\delta = 1/k$.
- Observations [1][2] indicate that duty cycles are smaller in systems with lower stellar masses. Comparison with the optimal feedback model suggests that the feedback strength is greater in elliptical galaxies than galaxy clusters. Then, if $H = kL_X$, this suggests that AGN heating episodes are proportionally more powerful in elliptical galaxies than in galaxy clusters. This is consistent with recent theories describing the steepening of the L_X-T relation for temperatures below 1-2 keV [5][6].
- The time-averaged black hole growth rate is $L_{X0}/[\eta c^2 \ln(4)]$, where η is the accretion efficiency and c is the speed of light.

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