PHYS2581 Foundations2A: EQM 6 solution

(a) $a = 4\pi\epsilon_0\hbar^2)/\mu Z e^2$. $\mu_{Fe} \approx m_e = \mu_H$ so $a_{Fe} = a_H/Z = a_H/26$ so its 26x smaller. [1 mark]

$E_1 = -13.6Z^2/n^2 = -9193$ eV [1 mark]

$n = 1 - 2 = -9193(1/1 - 1/4) = -6895$ eV

$n = 1 - 3 = -9193(1/1 - 1/9) = -8172$ eV

$n = 1 - 4 = -9193(1/1 - 1/16) = -8619$ eV [1 mark]

(b) reduced mass is $1848 \times 200 m_e^2/[(1848 + 200)m_e] \approx 180 m_e$

so $a \propto a_H/\mu$ so its 180x smaller [1 mark]

Energy is $E_1 \propto 1/\mu 1/a^2$ but $a \propto 1/\mu$ so $E_1 \propto \mu$ so its 180x bigger. [1 mark]

(c)

$$< p^2/2\mu >= \frac{1}{2\mu}\frac{1}{\pi a^3}\int\int\int e^{-r/a}\frac{-\hbar^2}{r^2}\frac{d}{dr}\Big[r^2\frac{d}{dr}(e^{-r/a})\Big]r^2\sin\theta dr d\theta d\phi$$

$$= \frac{1}{2\mu}\frac{4}{a^3}\int e^{-r/a}\frac{-\hbar^2}{r^2}\frac{d}{dr}\Big[r^2\frac{d}{dr}(e^{-r/a})\Big]r^2 dr$$

$$= \frac{-\hbar^2}{2\mu}\frac{4}{a^3}\int e^{-r/a}\frac{d}{dr}\Big[r^2\frac{d(e^{-r/a})}{dr}\Big]dr = \frac{-2\hbar^2}{\mu}\frac{1}{a^3}\int e^{-r/a}\frac{d}{dr}(r^2.-\frac{1}{a}e^{-r/a})dr$$

[2 marks]

$$= \frac{2\hbar^2}{\mu a^4}\int e^{-r/a}\frac{d}{dr}(r^2 e^{-r/a}) = \frac{2\hbar^2}{\mu a^4}\int e^{-r/a}(2re^{-r/a} - 1/a.r^2e^{-r/a}$$

$$= \frac{2\hbar^2}{\mu a^4}[2\int re^{-2r/a}dr - \frac{1}{a}\int r^2e^{-2r/a}dr$$

[1 mark]

$$= \frac{2\hbar^2}{\mu a^4}[2\frac{1!}{(2/a)^2} - \frac{1}{a}\frac{2!}{(2/a)^3}] = \frac{2\hbar^2}{\mu a^4}[\frac{a^2}{2} - \frac{a^2}{4}] = \frac{\hbar^2}{2\mu a^2}$$ [1 mark]

(d) $< V >= -\hbar^2/(\mu a)\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0}^{\infty}(\pi a^3)^{-1}e^{-2r/a}1/rr^2\sin\theta dr d\theta d\phi$

$== -\hbar^2/(\mu a^2\pi)4\pi\int_{r=0}^{\infty}e^{-2r/a}r dr$

$= -4\hbar^2/(\mu a^2)1!/(2/a)^2 = -\hbar^2/\mu = -2 < T >$