

## ELEMENTS OF QUANTUM MECHANICS - EXAMPLES CLASS 5

Q1: The electron in a Hydrogen atom is in a combined state

$$\psi(r, \theta, \phi) = \frac{1}{\sqrt{21}}(2\psi_{210} + \psi_{321} + 4\psi_{422})$$

What are the possible values and associated probabilities for a measurement of

- (a)  $L^2$
- (b)  $L_z$
- (c)  $H$  i.e. energy ( $E_n = -13.6/n^2$  eV)
- (d) If  $H$  is measured to be  $-13.6/9$  eV, what is the result of a subsequent measurement of  $H$  and  $L_z$ ? Do we get a deterministic answer if we now measure  $L_x$ ?

Hint  $[H, L^2] = [L^2, L_z] = 0$  but  $[L_z, L_x] = i\hbar L_y$

Q2: Any Hydrogen-like ion has a typical size scale  $a = 4\pi\epsilon_0\hbar^2/(\mu Ze^2)$ , reduced mass  $\mu = Mm/(M+m)$  wher  $M$  is the nuclear mass with charge  $+Ze$  and  $m$  is the mass with charge  $-e$ . This has energy levels  $E_n = -\hbar^2/(2\mu a^2 n^2)$ . In Hydrogen,  $E_1 = -13.6$ eV,  $a = a_H$  and  $\mu_H \approx m_e$ .

- (a) Calculate the energies in eV of the  $n = 3 - 2$  and  $n = 4 - 2$  (Balmer series,  $H\alpha$  and  $H\beta$ ) for Hydrogen from  $E_n = -13.6/n^2$ . Give your answer to 3 sig. figs.
- (b) The  $H\alpha$  'line' is actually 2 lines when seen at high resolution, separated by  $\Delta E = 4.5 \times 10^{-5}$  eV. We attributed this to electron spin, but some bright student suggests that instead its just due to there being some fraction of Hydrogen present as Deuterium, an isotope of Hydrogen with nuclear mass of  $\approx 2m_p$ . Calculate the ratio  $\mu_D/\mu_H$  using  $m_p/m_e = 1836$ . Write  $a_D/a_H$  as a function of  $\mu_D/\mu_H$  and hence find  $E_{n,D}/E_{n,H}$ , and the difference  $E_{n,D} - E_{n,H}$ . Calculate this in eV for the  $n = 3 - 2$  transition for deuterium. Can this explain the observed splitting?

Q3: Write down the expression for the momentum operator,  $p$ , in one dimension. Hence show that  $[H, p]\psi = i\hbar \frac{dV}{dx}\psi$  where  $H = p^2/2m + V(x)$