PHYS2581 Foundations2A: Q1 answer

i) normalisation $\int_0^L A^2 \sin^6(\pi x/L)dx = 1$ wolfram alpha gives $\int_0^L \sin^6(\pi x/L)dx = 5L/16$ so $A^2 = 16/(5L)$ so $A = \sqrt{16/(5L)} = 4/\sqrt{5L}$ [1 mark]

ii) $< x >= A^2 \int_0^L x \sin^6(\pi x/L)dx$ wolfram: $\int_0^L x \sin^6(\pi x/L)dx = 5L^2/32$ so $< x >= 16/(5L) \times 5L^2/32 = L/2$ [2 marks]

iii) $< p >= A^2 \int_0^L \sin^3(\pi x/L) \times -i\hbar d/dx(\sin^3 \pi x/L)dx$

$= -i\hbar A^2 \int_0^L (3\pi/L) \sin^5(\pi x/L) \cos(\pi x/L)dx$ [2 marks]

$= 0$ [1 mark]

iv) $< p^2 >= A^2 \int_0^L \sin^3(\pi x/L) \times -\hbar^2 d^2/dx^2(\sin^3 \pi x/L)dx$

$= -\hbar^2(16/5L) \times -(9\pi^2/16L)$ [2 marks]

$= 9\hbar^2\pi^2/5L^2$ [1 mark]

v) hence $< T >=< p^2 > /(2m) = 9\hbar^2\pi^2/(10mL^2)$ [1 mark]