Exam

Name_____

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

1) A wave function is given by 1) _____

$\psi(x) = 0 \qquad x < 0$
$\psi(x) = Ax\,(L - x) \qquad 0 \geq x \geq L$
$\psi(x) = 0 \qquad x > L$

The product of the normalization constant A and the quantity $L^{5/2}$ is equal to:
A) $\sqrt{24}$ B) $\sqrt{12}$ C) $\sqrt{30}$ D) $\sqrt{15}$ E) $\sqrt{20}$

2) The wave function for a particle must be normalizable because: 2) _____
A) the particle's charge must be conserved.
B) the particle's momentum must be conserved.
C) the particle cannot be in two places at the same time.
D) the particle's angular momentum must be conserved.
E) the particle must be somewhere.

Situation 40.1

A set of five possible wave functions is given, where L is a positive real number.

$\psi_1(x) = A\,e^{-x}$, for all x $\psi_2(x) = A\cos x$, for all x

$$\psi_3(x) = \begin{cases} Ae^{-x}, 0 \leq x \leq L \\ 0, \text{ all other } x \end{cases} \qquad \psi_4(x) = \begin{cases} A, -L \leq x \leq L \\ 0, \text{ all other } x \end{cases} \qquad \psi_5(x) = \begin{cases} Ax, x \geq L \\ 0, \text{ all other } x \end{cases}$$

3) In Situation 40.1, which of the five possible wave functions are normalizable? (There may be more than one correct choice.) 3) _____
A) $\psi_4(x)$ B) $\psi_1(x)$ C) $\psi_5(x)$ D) $\psi_2(x)$ E) $\psi_3(x)$

4) In Situation 40.1, the value of A required for normalization of $\psi_4(x)$ is: 4) _____
A) $1/\sqrt{L}$ B) $1/2L$ C) $1/\sqrt{2L}$ D) $1/L$ E) $1/L^2$

5) In Situation 40.1, the value of A required for normalization of $\psi_3(x)$ is: 5) _____

A) $\dfrac{2}{e^{2L}-1}$

B) $\sqrt{\dfrac{1}{e^{L}-1}}$

C) $\sqrt{\dfrac{2}{e^{2L}}}$

D) $\dfrac{1}{\sqrt{e^{L}}}$

E) $\sqrt{\dfrac{2}{e^{2L}-1}}$

Situation 40.2

A particle is confined to a one-dimensional box on the x-axis between $x = 0$ and $x = L$. The potential height of the walls of the box is infinite. The normalized wave function of the particle, which is in the ground state, is given by

$$\psi(x) = \sqrt{\frac{2}{L}} \sin\frac{\pi x}{L} \qquad 0 \le x \le L$$

6) In Situation 40.2, the probability of finding the particle between $x = 0$ and $x = L/3$, is closest to: 6) _____

A) 0.22 B) 0.26 C) 0.28 D) 0.20 E) 0.24

7) In Situation 40.2, the maximum probability per unit length of finding the particle is equal to: 7) _____

A) $1/\sqrt{L}$ B) $\sqrt{2}/\sqrt{L}$ C) $1/L$ D) $2/L$ E) $2/\sqrt{L}$

8) An electron is in an infinite square well that is 8.9-nm wide. The ground state energy of the electron is closest to: 8) _____

A) 0.0066 eV
B) 0.0085 eV
C) 0.0057 eV
D) 0.0076 eV
E) 0.0047 eV

9) An electron is in an infinite square well that is 9.6-nm wide. The electron makes the transition from the $n = 14$ to the $n = 11$ state. The wavelength of the emitted photon is closest to: 9) _____

A) 3400 nm B) 4100 nm C) 2800 nm D) 4700 nm E) 5300 nm

10) An electron is in an infinite square well that is 8.8-nm wide. The smallest value of the state quantum number n for which the energy level exceeds 100 eV is closest to: 10) _____

A) 145 B) 144 C) 142 D) 143 E) 146

11) An electron is bound in an infinite square-well potential on the x-axis. The width of the well is L and the well extends from $x = 0$ nm to $x = 7.3$ nm. In a given state, the normalized wave function of the electron is given by: 11) _____

$$\psi(x) = \sqrt{2/L}\ \sin(2\pi x/L)$$

The energy of the state is closest to:

A) 0.028 eV
B) 0.035 eV
C) 0.014 eV
D) 0.0071 eV
E) 0.021 eV

12) An electron is bound in an infinite square-well potential on the x-axis. The width of the well is L and the well extends from $x = 0$ nm to $x = 3.3$ nm. In a given state, the normalized wave function of the electron is given by: 12) _____

$$\psi(x) = \sqrt{2/L}\ \sin(2\pi x/L)$$

The probability per nm of finding the electron at $x = 1.65$ nm is closest to:

A) zero B) 1.1 C) 0.4 D) 1.3 E) 0.8

13) An electron is bound in an infinite square-well potential on the x-axis. The width of the well is L and the well extends from $x = 0$ nm to $x = 7.3$ nm. In a given state, the normalized wave function of the electron is given by: 13) _____

$$\psi(x) = \sqrt{2/L}\ \sin(2\pi x/L)$$

The probability per nm of finding the electron at $x = 0.30$ nm is closest to:

A) zero B) 0.0063 C) 0.013 D) 0.024 E) 0.018

14) An electron is in the ground state (lowest energy level) of an infinite well where its energy is 5.00 eV. In the next higher level, its energy would be closest to: 14) _____

A) 20.0 eV B) 1.25 eV C) 10.0 eV D) 15.0 eV E) 2.50 eV

Situation 40.3

A particle of mass m is bound in a square-well potential of finite height. The well is on the x-axis and extends from $x = -L/2$ to $x = +L/2$. There is only a single bound state, whose normalized wave function is given by:

$$\psi(x) = 1/\sqrt{L}\ \sqrt{\pi/4 + \pi}\ \exp(\pi/4)\ \exp(+\pi x/2L) \qquad x \le -L/2$$
$$\psi(x) = 1/\sqrt{L}\ \sqrt{2\pi/4 + \pi}\ \cos(\pi x/2L) \qquad -L/2 \le x \le +L/2$$
$$\psi(x) = 1/\sqrt{L}\ \sqrt{\pi/4 + \pi}\ \exp(\pi/4)\ \exp(-\pi x/2L) \qquad x \ge L/2$$

15) In Situation 40.3, denote the energy of the state by E. The ratio EmL^2/h^2 is closest to: 15) _____

A) 0.06 B) 0.03 C) 0.09 D) 0.12 E) 0.15

16) In Situation 40.3, the probability per unit length of finding the particle at the center of the well is closest to: 16) _____

A) $0.92/L$ B) $0.80/L$ C) $0.84/L$ D) $0.88/L$ E) $0.76/L$

17) In Situation 40.3, the smallest probability per unit length of finding the particle inside the well is closest to: 17) _____

A) $0.66/L$ B) $0.44/L$ C) $0.61/L$ D) $0.55/L$ E) $0.50/L$

18) In Situation 40.3, the probability of finding the particle outside the well is closest to: 18) _____

A) 0.24 B) 0.22 C) 0.20 D) 0.28 E) 0.26

19) An electron with kinetic energy 2.80 eV encounters a potential barrier of height $U_o = 4.70$ eV. If the barrier width is 0.4 nm, what is the probability that the electron will tunnel through the barrier? 19) _____

A) 1.1×10^{-1}

B) 1.4×10^{-2}

C) 2.8×10^{-2}

D) 5.5×10^{-2}

E) 1.4×10^{-1}

20) How does the probability of an electron tunneling through a potential barrier vary with the thickness of the barrier? 20) _____

A) It decreases inversely with thickness.
B) It decreases sinusoidally with thickness.
C) It decreases linearly with thickness.
D) It decreases exponentially with thickness.
E) It is independent of the barrier thickness.

21) Which of the following is an accurate statement concerning the simple harmonic oscillator? 21) _____

A) The potential energy varies linearly with displacement from equilibrium.
B) The spacing between energy levels increases with increasing energy.
C) The wave functions are sinusoidal functions.
D) The spacing between energy levels decreases with increasing energy.
E) The number of nodes of the wave function increases with increasing energy.

22) A low density beam of 60 eV electrons is directed at a potential barrier 130–eV high and 0.12–nm wide. The transmission coefficient is closest to: 22) _____

A) 7.1×10^{-6}

B) 1.4×10^{-4}

C) 2.8×10^{-5}

D) 2.8×10^{-4}

E) 1.4×10^{-5}

23) A low density beam of 8.0–eV electrons is incident on a potential barrier 15.0 eV in height and 0.80 nm in width. The fraction of the beam that is transmitted through the barrier is closest to: 23) _____

A) 5.0×10^{-7}

B) 1.7×10^{-9}

C) 1.7×10^{-6}

D) 3.3×10^{-5}

E) 2.5×10^{-8}

Situation 40.4

A free electron that has a kinetic energy of 3.0 eV is incident on a square–well potential. The depth of the well is 5.0 eV and the width is 4.0 nm.

24) In Situation 40.4, the wavelength of the wave function for the electron outside the well is closest to: 24) _____

A) 870 pm B) 710 pm C) 430 pm D) 550 pm E) 330 pm

25) In Situation 40.4, the wavelength of the wave function for the electron inside the well is closest to: 25) _____

A) 710 pm B) 330 pm C) 430 pm D) 550 pm E) 870 pm

Situation 40.5

The atoms in a nickel crystal vibrate as harmonic oscillators with an angular frequency of 2.3×10^{13} rad/s. The mass of a nickel atom is 9.75×10^{-26} kg.

26) In Situation 40.5, the effective interatomic force constant for nickel is closest to: 26) _____

A) 57 N/m B) 42 N/m C) 37 N/m D) 52 N/m E) 47 N/m

27) In Situation 40.5, the spacing of the vibrational energy levels of nickel is closest to: 27) _____

A) 0.019 eV B) 0.031 eV C) 0.023 eV D) 0.027 eV E) 0.015 eV

28) The lowest energy level of a certain quantum harmonic oscillator is 5.00 eV. The energy of the next higher level is closest to: 28) _____

A) 7.50 eV B) 10.0 eV C) 50.0 eV D) 15.0 eV E) 20.0 eV

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

29) The lowest energy level of a particle confined to a one–dimensional region of space with fixed dimension L is E_0 (i.e., a "particle in a box"). If an identical particle is confined to a similar region with fixed distance $(1/9)L$, what is the energy of the lowest energy level that the particles have in common? Express your answer in terms of E_0. 29) _____

30) Consider a particle in a box of width L and let the particle be in a state $n = 14$. What is the first value of x, larger than 0, where the probability of finding the particle is highest? 30) _____

31) An electron is bound in an infinite well of width 0.10 nm. If the electron is initially in $n = 8$ and falls to $n = 7$, find the wavelength of the emitted photon. 31) _____

Note: ($c = 3.0 \times 10^{8}$ m/s, $m_e = 9.11 \times 10^{-31}$ kg, and $h = 6.626 \times 10^{-34}$ j • s)

32) A lithium atom, mass 1.17×10^{-26} kg, is vibrating with simple harmonic motion in a crystal lattice, where the force constant k is 64.0 N/m. 32) _____

(a) What is the ground state energy of this system in eV?

(b) What would be the wavelength of the photon that could excite this system to the $n = 1$ level?

33) An electron in an infinite potential well makes a transition from the $n = 3$ level to the ground state (lowest energy level) and in so doing emits a photon of wavelength 20.9 nm. 33) _____

(a) What is the width of this well?

(b) What wavelength photon would be required to excite the electron from its original level to the next higher one?

(electron mass = 9.11×10^{-31} kg, $\hbar = 1.055 \times 10^{-34}$ J • s, $h = 6.626 \times 10^{-34}$ J • s)

1) C
2) E
3) A, E
4) C
5) E
6) D
7) D
8) E
9) B
10) B
11) A
12) A
13) E
14) A
15) B
16) D
17) B
18) D
19) B
20) D
21) E
22) B
23) B
24) B
25) C
26) D
27) E
28) D
29) $81E_{\ o}$
30) $L/28$
31) 2.2×10^{-9} m
32) (a) 2.44×10^{-2} eV
(b) 2.55×10^{-5} m
33)