ELEMENTS OF QUANTUM MECHANICS - EXAMPLES CLASS 6

Q1: This gives you practice in the new Dirac Notation! Consider

$$\frac{d < Q >}{dt} = \frac{d}{dt} < \psi|Q\psi > = < \frac{\partial \psi}{\partial t}|Q\psi > + < \psi|\frac{\partial Q}{\partial t}\psi > + < \psi|Q\frac{\partial \psi}{\partial t} >$$

(remember the d/dt goes to $\partial/\partial t$ when you take it INSIDE the integral)

(a) We know that $H\psi = i\hbar\partial\psi/\partial t$ so $\partial\psi/\partial t = -i/\hbar H$. Substitute this in your expression above and rearrange using the fact that H is hermitian (so $< g|Hf > = < Hg|f >$ to show that

$$\frac{d < Q >}{dt} = \frac{i}{\hbar} < [H, Q] > + \Big\langle \frac{\partial Q}{\partial t} \Big\rangle$$

(b) Set $Q = x$ and show that the expression in (a) above implies that $d < x > /dt = \frac{<p>}{m}$ given that $[H, x] = -\frac{i\hbar}{m}p$. This is what we expect from classical physics (Ehrenfest theorem - quantum eventually gives the same answers as classical)

(c) Set $Q = xp$ and show that the expression in (a) above implies that $d < xp > /dx = < T > = 1/2 < xdV/dx >$ given that $[H, xp] = -(i\hbar/m)p.p + x.i\hbar dV/dx$

(d) In steady state $d < xp > /dx = 0$. Find a relation for $< T >$ in terms of dV/dx. This is the VIRIAL THEOREM in 1D cartestian coordinates and its useful as in general calculating $< T >$ is hard as its a second order differential operator, whereas calculating $< dV/dx >$ is easier.