ELEMENTS OF QUANTUM MECHANICS - EXAMPLES CLASS 2

1. $\Psi(x, t = 0) = Ax(L - x)$ in the infinite square well potential for $0 < x < L$ and zero elsewhere, where $A = \sqrt{30/L^5}$. Eigenfunctions of this potential are $\psi_n(x) = \sqrt{2/L} \sin n\pi x/L$, corresponding to energy $E_n = n^2\pi^2\hbar^2/(2mL^2) = n^2 E_1$.

   i) $\Psi = \sum_n c_n \psi_n$ so $c_n = \int \psi_n^* \Psi dx$. Use this to calculate the probability that the systems is measured to have energy E_n. (REMEMBER $\cos(n\pi) = \pm 1$ depending on whether n is odd or even)

   ii) Write down the first 3 terms explicitally, and hence show that the general form for even and odd n is $c_n = 0$ for even n, $c_n = 8\sqrt{15}/(n^3\pi^3)$ for odd.

   iii) What is the probability (3 sig figs) that the system is measured to be in the ground state?

   iv) Write down an infinite sum for $< E >$ in terms of E_1. calculate it from $\sum_{oddn} 1/n^4 = \pi^4/96$. Why is this bigger than E_1?

   v) calculate it the slow way from $< E >= \int \Psi^* H \Psi dx$ (Hint: $H = -(\hbar^2/2m)d^2/dx^2$ as $V = 0$ in the well) and show it is the same.

   Useful Integrals (c is an integration constant)

$$\int x \sin(ax) dx = \frac{\sin(ax) - ax\cos(ax)}{a^2} + c$$

$$\int x^2 \sin(ax) dx = \frac{(2 - a^2x^2)\cos(ax) + 2ax\sin(ax)}{a^3} + c$$