PHYS2581 Foundations2A: QM2 7

Hydrogen is subject to a perturbation from an external electric field E_{ext} in the z direction, so $H' = eE_{ext}r\cos\theta$

(a) The unperturbed $n = 1$ level in Hydrogen is non-degenerate (ignoring spin) and has wavefunction $\psi^0_{100} = (\pi a^3)^{-1/2}e^{-r/a}$. Show that there is no first order correction to the ground state energy i.e. $E^1_1 = \int \psi^{0*}_{100}H'\psi^0_{100}dV = 0$

(b) The unperturbed $n = 2$ level in Hydrogen is 4-fold degenerate, with energy eigenfunctions

$$|1>= \psi^0_{200} = \sqrt{\frac{1}{2\pi a}}\,\frac{1}{2a}(1-\frac{r}{2a})e^{-r/2a}$$

$$|2>= \psi^0_{211} = -\frac{1}{\sqrt{\pi a}}\,\frac{r}{8a^2}e^{-r/2a}\sin\theta e^{i\phi}$$

$$|3>= \psi^0_{210} = \frac{1}{\sqrt{2\pi a}}\,\frac{r}{4a^2}e^{-r/2a}\cos\theta$$

$$|4>= \psi^0_{21-1} = \frac{1}{\sqrt{\pi a}}\,\frac{r}{8a^2}e^{-r/2a}\sin\theta e^{-i\phi}$$

Use degenerate perturbation theory to determine the first order correction, E^1_2, to the $n = 2$ level by writing down a 4x4 matrix equation with terms $W_{ij} =< i|H'j >$ for $i, j = 1, 2, 3, 4$. [Hint: do the angle integrals first as many of these are zero which saves you the trouble of doing the radial integral!]

(c) Solve the matrix to determine all possible values of E^1_2. Into how many different energy levels does $n = 2$ split?

(d) Write down the wavefunctions $\chi = \alpha|1 > +\beta|2 > +\gamma|3 > +\delta|4 >$ for which we could have used non-degenerate perturbation theory.

Useful integrals (look up anything else you need):

$$\int_0^\infty x^p e^{-qx}dx = \frac{p!}{q^{p+1}}$$