PHYS2581 Foundations2A: QM2.5

The $n = 1$ and $n = 2$ states of Hydrogen have radial wavefunctions

$$R_{10} = 2a^{-3/2}e^{-r/a}$$

$$R_{20} = \frac{1}{\sqrt{2}}a^{-3/2}\left(1 - \frac{r}{2a}\right)e^{-r/2a}$$

$$R_{21} = \frac{1}{\sqrt{24}}a^{-3/2}\frac{r}{a}e^{-r/2a}$$

so the full $\psi_{nlm} = R_{nl}Y_{lm}$.

(a) For the n=1 state, calculate the probability that the electron will be found inside the proton, assuming that the energy eigenfunction above is valid inside the proton and that the proton radius in units of the Bohr radius is $r_p/a = 1.9 \times 10^{-5}$ [Hint: $r_p << a$ so $e^{-r/a} \approx 1$]

(b) Find the average (expectation value) of electron-proton separation $< r >$ for all possible ψ_{2lm}. Does $< r >$ depend on l? Does $< r >$ depend on m?

(c) Calculate the probability that the electron is found inside the proton for n, l=$2, 0$ and n, l=$2, 1$. The difference tells you about the different behaviour as $r \to 0$ of the radial proability density for l=1 and 0 (look at the 3D orbital picture linked from lecture 12)

Useful integrals

$$\int_0^\infty x^p e^{-qx} dx = \frac{p!}{q^{p+1}}$$