PHYS2581 Foundations2A: Q2

The energy eigenfunctions of a particle in an infinite square well from $0 \leq x \leq L$ are $\psi_n(x) = \sqrt{\frac{2}{L}} \sin(n\pi x/L)$. A particle in the well has an initial wave function which is NOT itself an energy eigenfunction,

$$\Psi(x, t=0) = A \sin^3(\pi x/L) \quad \text{where } A = \left(\frac{16}{5L}\right)^{1/2}$$

Look up any integrals you need at

http://www.wolframalpha.com/

i) $\Psi(x,0)$ can be decomposed into a weighted sum of energy eigenfunctions so that $\Psi(x,0) = \sum_n c_n \psi_n(x)$ where $c_n = \int \psi_n^*(x)\Psi(x,0)dx$. Use Wolfram alpha to do this integral for each of n=1,2,3 and 4 [5 marks]

ii) Use the trigonometic identity $\sin^3 x = \frac{1}{4}[3\sin x - \sin(3x)]$ to show that $\Psi(x,0) = 1/\sqrt{10}(3\psi_1 - \psi_3)$ hence determine c_n analytically and check your answers above. [2 marks]

iii) What is the probability that a measurement of energy gives the value E_1? [1 mark]

iv) What is the expectation value of the energy $< H >= \sum_n c_n^2 E_n$? give your answer in terms of E_1 (so write $E_n = n^2 E_1$). [1 mark]

v) Write down the fully time dependent wavefunction $\Psi(x,t)$ [1 mark]