PHYS2581 Foundations2A: QM2.5 answers

(a) prob find from $r = 0$ to rp

$$\int_0^{rp} 4a^{-3}e^{-2r/a}r^2 dr \approx \frac{4}{a^3}[r^3/3]_0^{rp} = \frac{4}{a^3}\frac{rp^3}{3} = \frac{4}{3}1.9 \times 10^{-15} = 9.1 \times 10^{-15}$$

(b) need n, l, m of (2,0,0), (2,1,0), (2,1,1), (2,1,-1)

$$< r >_{nlm} = \int\int\int rR_{nl}^2 Y_{lm}^* Y^{lm} r^2 \sin\theta d\theta d\phi = \int r^3 R_{nl}^2 dr$$

so this can only depend on n and l not m

$$< r >_{200} \int r^3 \frac{1}{2a^3}\Big(1 - \frac{r}{2a}\Big)^2 e^{-r/a} dr = \frac{1}{2a^3}\Big(1 - \frac{r}{a} + \frac{r^2}{4a^2}\Big) e^{-r/a} dr$$

$$= \frac{1}{2a^3}\Big[\int r^3 e^{-r/a} dr - \frac{1}{a}\int r^4 e^{-r/a} dr + \frac{1}{4a^2}\int r^5 e^{-r/a} dr\Big]$$

$$= \frac{1}{2a^3}\Big[\frac{3!}{(1/a)^4} - \frac{1}{a}\frac{4!}{(1/a)^5} + \frac{1}{4a^2}\frac{5!}{(1/a)^6}\Big]$$

$$= \frac{1}{2a^3}(6a^4 - 24a^4 + 30a^4) = \frac{a}{2}12 = 6a$$

$$< r >_{21m} = \int r^3 \frac{1}{24a^3}\frac{r^2}{a^2} e^{-r/a} dr = \frac{1}{24a^5}\int r^5 e^{-r/a} dr = \frac{1}{24a^5}\frac{5!}{(1/a)^6}$$

$$= \frac{a}{24}(5.4.3.2) = 5a$$

higher l need smaller radii

(c) nl=2,0 prob find from $r = 0$ to rp

$$\int_0^{rp} \frac{1}{2} a^{-3} \Big(1 - \frac{r}{2a}\Big)^2 e^{-r/a} r^2 dr \approx \frac{1}{2a^3} \int \Big(1 - \frac{r}{2a}\Big)^2 r^2 dr$$

$$= \frac{1}{2a^3} \int \Big(r^2 - \frac{r^3}{a} + \frac{r^4}{4a^2}\Big) dr \approx 1/(2a^3)(rp^3/3) = (rp/a)^3/6 = 1.1 \times 10^{-15}$$

n,l=2,1 prob find from $r = 0$ to rp

$$\int_0^{rp} \frac{1}{24a^3} \Big(\frac{r}{a}\Big)^2 e^{-r/a} r^2 dr \approx \frac{1}{24a^5} (rp^5/5)$$

$= (rp/a)^5/(5.24) = 2.0 \times 10^{-26}$