

ELEMENTS OF QUANTUM MECHANICS - EXAMPLES CLASS 1

1. $\Psi(x, t = 0) = N \sin(\pi x/L)$ for $0 < x < L$ and zero elsewhere

i) Show that $N = \sqrt{\frac{2}{L}}$. Calculate the probability to find the electron in the interval $0 < x < L/4$. Give your answer to 3 sig. figs.

ii) Calculate $\langle x^2 \rangle = \int \Psi^* x^2 \Psi dx$

iii) Calculate $\langle p^2 \rangle$, and hence calculate kinetic energy $\langle T \rangle = \langle p^2 \rangle / 2m$

iv) Use physical arguments to just write down $\langle p \rangle$ and $\langle x \rangle$ (or do the integral and then figure out how you should have known the answer in advance!)

v) $(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2$ and $(\Delta p)^2 = \langle p^2 \rangle - \langle p \rangle^2$. Hence calculate $\Delta x \Delta p$ for this system and relate your answer to the Heisenberg Uncertainty principle $\Delta x \Delta p \geq \hbar/2$

vi) Calculate $\langle xp \rangle = \int \Psi^* xp \Psi dx$

vii) Calculate $\langle px \rangle = \int \Psi^* px \Psi dx$

viii) hence show that $\langle xp \rangle - \langle px \rangle = i\hbar$

Useful Integrals

$$\int \sin^2(ax) dx = \frac{x}{2} - \frac{\sin(2ax)}{4a}$$

$$\int \sin(ax) \cos(ax) dx = -\frac{\cos^2(ax)}{2a}$$

$$\int x \sin(ax) \cos(ax) dx = \frac{\sin(2ax) - 2ax \cos(2ax)}{8a^2}$$

$$\int x \sin^2(ax) dx = -\frac{\cos(2ax)}{8a^2} - \frac{x \sin(2ax)}{4a} + \frac{x^2}{4}$$

$$\int x^2 \sin^2(ax) dx = -\frac{x \cos(2ax)}{4a^2} - \frac{(2a^2x^2 - 1) \sin(2ax)}{8a^3} + \frac{x^3}{6}$$