PHYS2581 Foundations2A: EQM 6

Hydrogen-like ions have orbitals with typical size scale $a = 4\pi\epsilon_0\hbar^2/(\mu Ze^2)$, reduced mass $\mu = Mm/(M+m)$ wher M is the nuclear mass with charge +Ze and m is the mass with charge -e. This has energy levels $E_n = -\hbar^2/(2\mu a^2 n^2)$. In Hydrogen, $E_1 = -13.6eV$, $a = a_H$ and $\mu_H \approx m_e$.

(a) Calculate the typical size scale in units of a_H for Hydrogen-like iron ($Z = 26$, mass of the nucleus is $55.8m_p$). Calculate the energies (in eV) of E_1 and the $n = 1-2$, $n = 1-3$ and $n = 1-4$ transitions (1st three Lyman series) for this ion. [3 marks]

(b) Calculate the typical size scale in units of a_H and energy E_1 (in units of E_1 for hydrogen) for a bound state made from a proton and muon (muon charge is $-e$, mass is $200m_e$). [2 marks]

(c) Any hydrogen-like ion has a ground state wavefunction of $\psi_{100} = (\pi a^3)^{-1/2}e^{-r/a}$. This state has $l = 0$ so the kinetic energy operator simplifies to

$$T = \frac{p^2}{2\mu} = -\frac{\hbar^2}{2\mu r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right)$$

calculate $< T >$ (leave your answer in terms of μ and a). [4 marks]

(d) Compare $< T >$ in (c) above with $< V >$ for this state, where $V(r) = -Ze^2/(4\pi\epsilon_0 r) = -\hbar^2/(a\mu r)$ [1 mark]

Useful Integrals (but look up anything else you need on wolfram alpha)

$$\int_0^\infty x^p e^{-qx} dx = \frac{p!}{q^{p+1}}$$