(a) $p = -i\hbar d/dx$ [1 mark]

$\langle p \rangle = \int \Psi^* p \Psi dx$ [1 mark]

$= -i\hbar\sqrt{\frac{2}{L}}\int_0^L sin(\pi x/L)d/dxsin(\pi x/L)dx$

$= -i\hbar\sqrt{\frac{2}{L}}\int_0^L \sin(\pi x/L)\frac{\pi}{L}\cos(\pi x/L)dx = 0$ [1 mark]

its a standing wave [1 mark]

(b) 1 mark for potential

1 mark for ground

1 mark for first excited state

the peak is shifted away from the midpoint towards the origin as there is more probability that it can be in the middle energy barrier than at the edges due to leakage from the other well [1 mark]

(c) $E = \hbar\omega_x(n_x + 1/2) + \hbar\omega_x(n_y + 1/2) + 2\hbar\omega_x(n_z + 1/2)$

$= \hbar\omega_x(n_x + n_y + 2n_z + 2)$ [1 mark]

ground. $n_z = n_y = n_z = 0$ so $E = 2\hbar\omega_x$ degeneracy 1 [1 mark]

first excited $n_z = 1$, $n_y = n_z = 0$

or $n_z = 0$, $n_y = 1$ $n_z = 0$ (not $n_z = 1$ as this is next level up) so degeneracy is 2 and energy $E = 3\hbar\omega_x$ [1 mark]

maximize degeneracy if $\omega_x = \omega_y = \omega_z$. minimize if not comensurate [1 mark]

(d) prob is $\psi^*_{000}\psi_{000}dV = \psi^*_{000}\psi_{000}r^2\sin\theta drd\theta d\phi$ [1 mark]

prob within dr of r means integrate over angles so

$= \int_0^\pi\int_0^{2\pi}(\pi a^3)^{-1}e^{-2r/a}r^2\sin\theta drd\theta d\phi$ [1 mark]

$= 4\pi/(\pi a^4)e^{-2r/a}r^2dr = (4/a^3)r^2e^{-2r/a}$ [1 mark]

peaks when $dD(r)/dr = 0$ so $2re^{-2r/a} - 2/ar^2e^{-2r/a} = 0$ and $r = a$. [1 mark]

(a) $c_1 = \frac{4\sqrt{15}}{\pi^3}(1 - \cos(\pi)) = 8\sqrt{15}/(\pi^3)$ [1 mark]

$c_2 = \frac{4\sqrt{15}}{8\pi^3}(1 - \cos(2\pi)) = 0$ [1 mark]

$c_3 = \frac{4\sqrt{15}}{27\pi^3}(1 - \cos(3\pi)) = 8\sqrt{15}/(27\pi^3)$ [1 mark]

general form is $c_n = 0$ for even n, $c_n = 8\sqrt{15}/(n^3\pi^3)$ for odd [1 mark]

all even n will be zero as these are all antisymmetic about the midpoint whereas the wavefunction to be described is symmetic. [2 marks]

(b) $< E >= \sum_n c_n^2 E_n$ [1 mark]

$= \sum_{oddn} 64.15/(n^6\pi^6)n^2 E_1$ [1 mark]

$= \sum_n (64.15/\pi^6)E_1/n^4 = (64.15/96)E_1/\pi^2 = 10E_1/\pi^2$ [1 mark]

no single measurement can give this as its not one of E_n. [1 mark]