PHYS2581 Foundations2A: QM 4

i) A rigid rotator has energy levels $E_l = l(l+1)\hbar^2/(2I)$ where I is the moment of inertia and l is the azimuthal quantum number. Write down explicitally the possible values for the magnetic quantum number m for $l = 3, 4$ and 5. What is the energy and degeneracy of each level? [1 mark]

ii) The energy eigenfunction for the rigid rotator above in the $l = 2$, $m = -1$ state is

$$Y_{2,-1} = \left(\frac{15}{8\pi}\right)^{1/2} \sin\theta\cos\theta e^{-i\phi}$$

Write down the probability dP of finding the electron with this wavefunction in a region $dV = \sin\theta d\theta d\phi$ about position θ, ϕ. Integrate this over ϕ to get the probability density per unit θ. At what values(s) of θ does this have a maximum? Evaluate your answer in degrees to 3 significant figures.[3 marks]

Evaluate $< \theta >$ (Notice that $< \theta >$ and the most probable value of θ are NOT the same thing in general!) and $< \cos\theta >$ [2 marks]

What is the probability of finding the electron in the region $0 < \theta < \pi/3$. Evaluate your answer to 3 significant figures. [1 mark]

iii) Demonstrate explicitly that $Y_{2,-1}$ is an eigenfunction of the the angular momentum operators

$$\mathbf{L}^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right] \text{ and } L_z = -i\hbar\frac{\partial}{\partial\phi}$$

but is *not* an eigenfunction of

$$L_x = i\hbar\left(\sin\phi\frac{\partial}{\partial\theta} + \cot\theta\cos\phi\frac{\partial}{\partial\phi}\right)$$

[3 marks]

Useful integrals:

Use an online integrator such as http://www.wolframalpha.com/
where you can evaluate definite integrals by typing e.g.
integrate cos x from 0 to pi