PHYS2581 Foundations2A: QM3

QM3: A potential well has $V = \infty$ for $x < 0$, $V = 0$ for $0 < x < L$ and $V = V_0$ for $x > L$. Use Wolfram alpha for equation solving and integrals in parts b and c.

a) Solve the time independent Schroedinger equation for bound states ($E < V_0$) of this system. Show that this leads to the condition $kL \cot kL = -\rho L$ where $k^2 = 2mE/\hbar^2$ and $\rho^2 = 2m(V_0 - E)/\hbar^2$. [2 marks]

b) Show that this condition can be rewritten as $\sqrt{(z_0/z)^2 - 1} = -\cot z$ for $z = kL$ and $z_0^2 = 2mV_0L^2/\hbar^2$. Solve this for the specific case of $z_0 = 6$ using Wolfram alpha. Quote your answers for the solutions z_n to 3 significant figures. How many bound states are there, and what are their energies E_n in units of $\hbar^2/(2mL^2)$? [4 marks]

c) Determine the values of k and ρ in terms of L for the ground state for $z_0 = 6$. Hence normalise the ground state wavefunction. [4 marks]