The 1D infinite square well potential, where $V(x) = 0$ for $0 < x < L$ and ∞ elsewhere, has energy eigenfunctions $\psi_n(x) = \sqrt{2/L}\sin n\pi x/L$, corresponding to energy $E_n = n^2\pi^2\hbar^2/(2mL^2) = n^2 E_1$

An electron in this potential has $\Psi(x, t = 0) = Ax(L - x)$ where $A = \sqrt{30/L^5}$.

(a) $\Psi = \sum_n c_n\psi_n$, where $c_n = \int \psi_n^*\Psi dx$. show that $c_n = \frac{4\sqrt{15}}{n^3\pi^3}(1 - \cos(n\pi))$ [5 marks]

(b) Write down the first 3 terms explicitally, and use these to give a general form for odd n. What is the general form for even n? Give a physical explanation for your answer. [3 marks]

(c) Write down an infinite sum for $< E >$ in terms of E_1, and calculate this given that $\sum_{oddn} \frac{1}{n^4} = \frac{\pi^4}{96}$. Can any measurement of E give this value? [2 marks]

Useful Integrals

$$\int_0^L x\sin(n\pi x/L)dx = \frac{L^2}{n^2\pi^2}[\sin(n\pi) - n\pi\cos(n\pi)]$$

$$\int_0^L x^2\sin(\pi x/L)dx = \frac{L^3}{n^3\pi^3}[(2 - n^2\pi^2)\cos(n\pi) + 2n\pi\sin(n\pi) - 2]$$