PHYS2581 Foundations2A: Q2 answer

i) $c_1 = \int_0^L \frac{4}{\sqrt{5L}} \sin^3(\pi x/L)\sqrt{\frac{2}{L}} \sin \pi x/L dx = \frac{4}{L}\sqrt{\frac{2}{5}} \int_0^L \sin^4(\pi x/L) dx$ wolfram: $\int_0^L \sin^4(\pi x/L)dx = 3L/8$ so $c_1 = \frac{4}{L} \times \sqrt{\frac{2}{5}} \times \frac{3L}{8} = \frac{3}{\sqrt{10}}$ [2 marks]

$c_2 = \sqrt{\frac{32}{5L^2}} \int_0^L \sin^3(\pi x/L) \sin 2\pi x/L dx = 0$ in wolfram alpha [1 mark]

$c_3 = \frac{4}{L}\sqrt{\frac{2}{5}} \int_0^L \sin^3(\pi x/L) \sin 3\pi x/L dx = \frac{4}{L} \times \sqrt{\frac{2}{5}} \times -L/8 = -1/\sqrt{10}$ [1 mark]

$c_4 = \frac{4}{L}\sqrt{\frac{2}{5}} \int_0^L \sin^3(\pi x/L) \sin 4\pi x/L dx = 0$ [1 mark]

ii)

$$\Psi(x,0) = A\sin^3(\pi x/L) = \frac{A}{4}(3\sin \pi x/L - \sin 3\pi x/L)$$

$$= \frac{1}{\sqrt{5L}}(3\sin \pi x/L - \sin 3\pi x/L) = \frac{1}{\sqrt{5L}}\sqrt{\frac{L}{2}}(3\sqrt{\frac{2}{L}}\sin \pi x/L - \sqrt{\frac{2}{L}}\sin 3\pi x/L)$$

$$= \frac{1}{\sqrt{10}}(3\psi_1 - \psi_3)$$

[1 mark]

hence $c_1 = 3/\sqrt{10}$ and $c_3 = -1/\sqrt{10}$ and all the rest of the $c_n = 0$ [1 mark]

iii) prob $E_1 = |c_1|^2 = 9/10$ [1 mark]

iv) $< H >=< E >= \sum_n c_n^2 E_n = 9/10E_1 + 1/10E_3 = 9/10E_1 + 1/10 \times 9E_1 = 18/10E_1$ [1 mark]

v) $\Psi(x,t) = \frac{1}{\sqrt{10}}(3\psi_1 e^{-iE_1 t/\hbar} - \psi_3 e^{-iE_3 t/\hbar})$ [1 mark]