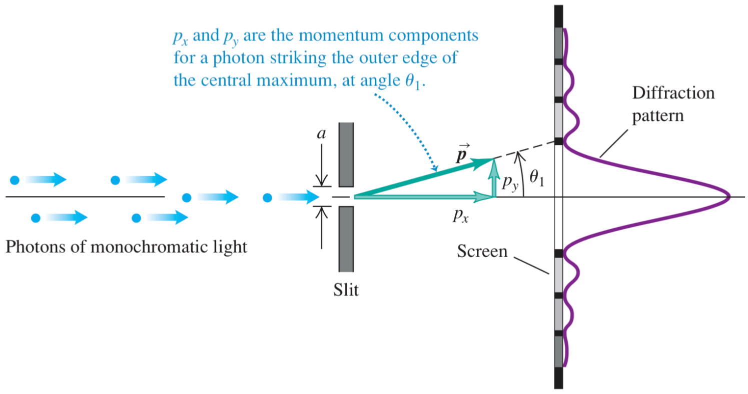


We saw last lecture that we get our classic double slit interference pattern

slit width a, and separation d gives intensity $I \propto \cos^2(\pi\theta d/\lambda)\text{sinc}^2(\pi\theta a/\lambda)$ where $\text{sinc}^2 x = \sin x/x$

but its made from individual detection events. Those events happen AT RANDOM within the expected wave intensity interference pattern! We can't predict exactly where an individual photon will be detected, we can only predict a PROBABILITY where the individual photon will land using the wave intensity.

1.5 uncertainty on momentum - localise a particle

This has profound consequences in terms of an ultimate limit on our knowledge of position! Photons are not like particles in Newtonian mechanics if they sometimes behave like waves. We can illustrate this by the uncertainty principle for a single slit. Suppose we have only a single slit, with width a. Then a wave with wavelength λ is diffracted into a pattern $y(\theta) \propto \sin(x)/x$ where $x = \pi a\theta/\lambda$.

We don't know where any individual photon will end up on the screen. We only know that after a while, we'll get the intensity pattern predicted by the wave amplitude squared!

Since position on the screen is uncertain to some level, the photon *momentum* must also have some level of uncertainty. The way to see this is that the photons come along with no momentum in the y direction. Yet they are diffracted to positions where $y > a$. On average the whole pattern is symmetric about 0, but any individual photon has to get to its y position so it must have some momentum in the y direction.

So lets just think about the first maximum as 85% of photons end up in this region. Peak is when $x \to 0$, $\sin x/x \to x/x \to 1$. minima when $\sin(x) = 0$ so $x = n\pi$ for $n \neq 0$. The first maximum is from $n = -1$ to $n = 1$ corresponding to angles $-\theta_1 \to \theta_1$. This gives $x = \pi = \pi\theta_1 a/\lambda$ so $\theta_1 = \lambda/a$.

So 85% of the photons go through a deflection $|\theta| \sim \theta_1$

Then momentum $p_y/p_x \approx \tan\theta_1 \approx \theta_1$. so $p_y \approx p_x\theta_1 = p_x\lambda/a$

But $p_x = h/\lambda$ so $p_y \leq h/a$.

so there is an inherent uncertainty in momentum, of order $\Delta p_y = h/a$ which comes by constraining the photons to a small slit in this direction $\Delta y = a$. So we have a real uncertainty in where any individual particle ends up of $\Delta y \Delta p_y \sim h$.

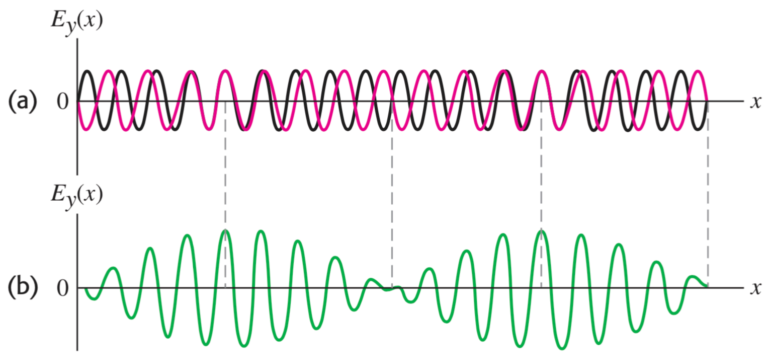
More careful consderation of factors of 2π gives an absolute limit on knowlegde of any individual photon of $\Delta y \Delta p_y \sim \hbar/2$.

So quantum particles like photons are not like classical particles. there is an irreducible limit to the accuracy at which their momentum and position can be calculated in any direction. The better we know position along any axis, the more uncertain is the value of momentum along that same axis.

there is no classical description in terms of just particles or just waves which works for understanding the behaviour of light.

1.6 Uncertainty on position - localise a wave

Suppose we have an EM wave, travelling from left to right with velocity $c = \lambda f = \omega/k$ where k is wavenumber $k = 2\pi/\lambda$ and $\omega = 2\pi f$ is angular frequency.



This has a single unique value of wavelength λ but can be *anywhere* in space. There is infinite uncertainty on its position! But we know now that photons have $p = h/\lambda = \hbar k$ so there is a one to one relation between wavelength and momentum. A plane wave has no uncertainty on its momentum, but is extended over all space so there is infinite uncertainty on its position.

We want to localise the wave in space - have most of its amplitude over a small range in position. And we can do this by adding in another wave of different wavelength so that we get interference 'beats'

e.g for 2 waves propagating from left to right along the x direction, with different wavelength i.e. different values of $k = p_x/\hbar$ and E then we get interference 'beats'

$y(x,t) = A_1 \sin[(p_{1x}x - E_1 t)/\hbar] + A_2 \sin[(p_{2x}x - E_2 t)/\hbar]$

Lets just look at a single time, say $t = 0$, and let $A_2 - = -A_1 = 1$ for simplicity. This is

$y(x, t = 0) = [\sin(p_{1x}x/\hbar) + \sin(p_{2x}x/\hbar)] = 2\cos[(p_{1x} - p_{2x})x/2\hbar]\sin[(p_{1x} + p_{2x})x/2\hbar]$

The beat pattern means there is larger amplitude at some points than others, so there is a bigger probability to find the photon at these points than at the others. We have partially localised the wave on the beat size scale of $\Delta x \sim \lambda_{beat}/2$. $\lambda_{beat} = 2h/(p_{1x} - p_{2x})$

BUT we did this by allowing different momenta components. we no longer have a well defined momentum - we can have either p_{1x} or p_{2x}. so there is a spread in momentum $\Delta p_x = (p_{2x} - p_{1x})/2$ about the mean value $(p_{1x} + p_{2x})/2$

so we can combine them and calculate the typical uncertainty on position and momentum by

$\Delta x \Delta p_x = 2h/[(p_{1x} - p_{2x})] \times (p_{2x} - p_{1x})/2 = h$

Waves have intrinsic uncertainty on position and momentum - it is NOT fundamentally a measurement issue, its a fundamental issue of waves, that they are spread out infinitely in space if they have a unique momentum, and the way to localise the wave is to add waves of different momenta. The better we localise in space, the wider the range of momenta needed, until we can get a unique point in space only from an infinite range of momenta.

2 Particles as waves

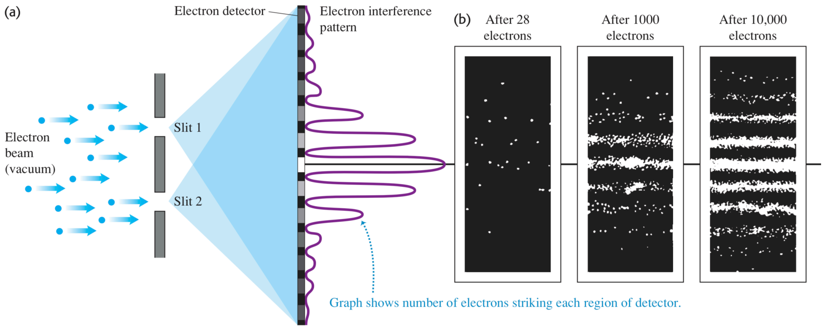
2.1 double slit experiement

if photons are light behaving as a particle rather than a wave, then what about particles? can particles behave like waves?

yes indeed, this is exactly what we see. Electrons - little bits of matter - give a diffraction pattern in the double slit experiment. this is direct confirmation that they have wave-like properties. Its EXACTLY like photons, with the interference pattern emerging from the build up of many INDIVIDUAL particle detections on the screen. Only this time we expected the individual detection events but NOT the interference pattern!

Because there is an interference pattern we know there must be a wavelength, and we can calculate this from the spacings of the maxima and minima

eg electrons with KE of 54eV produce first maximum at an angle of 50° when scattered through Ni crystal with spacing $d = 0.215$nm which was measured from X-ray diffraction



$m\lambda = d\sin\theta$ so $\lambda = 0.165$nm.

If this were light we'd have $\lambda = h/p$ - but now we have particles so $p = mv$

$p^2/2m = KE$ so $54 \times 1.6 \times 10^{-19} \times 2 \times 9.1 \times 10^{-31} = p^2$ and $p = 3.9 \times 10^{-24}$ kg m/s $\lambda = h/p = 1.66 \times 10^{-10}$ m i.e. 0.166nm as above.

so on small enough scales, electrons act as waves not particles. which means they are like light - neither a wave nor a particle but having aspects of both.

Example: Calculate the de broglie wavelength for electrons with energy of 1eV and 1keV

(we won't go higher than this as we'd need to think about relativity as we are getting close to $v = c$.)

i) $\lambda = h/p$ - but given energy so use $E = p^2/2m$ or $p = \sqrt{2mE}$ and 1eV=1.6×10^{-19} J so $p = 5.4 \times 10^{-25}$ kg/m/s and $\lambda = 1.22 \times 10^{-9}$ ie 1.22nm

ii) KE=1 keV - so should be $\sqrt{10^3}\times$ smaller ie 3.8×10^{-11}m=0.038nm

According to de Broglie, *all* particles have wave-particle duality, they all have wavelength $\lambda = h/p$. Smaller mass particles are more obviously wavy! but bigger ones have smaller wavelength so their waviness is only apparent on smaller scales.

example A proton is accelerated though a potential of 54V - what is its wavelength? compare this to an electron accelerated through the same potential

$\lambda = h/p$ and we are non-relativisitic so $K = eV = p^2/2m$

for electrons $K = 1.6 \times 10^{-19} 54 = 8.6 \times 10^{-18}$ so $p = \sqrt{2mK} = 3.9 \times 10^{-24}$ kg/m/s so $\lambda = h/p = 1.66 \times 10^{-10}$m

for protons, m is 1840x bigger so p is 42$\times$ bigger and λ is 42x smaller.

In the macroscopic world 'things' are either waves or particles but not both. Electrons and photons are NEITHER like classical waves NOR like classical particles. the thing they are most like is each other we don't have good ways to picture this yet though we do have excellent and very powerful ways to calculate their behaviour.

Experiments show us that the wave pattern gives us a statistical distribution - the intensity (amplitude squared) gives us the *probability* that any individual event will be detected at each position on the screen. we *can't* predict individual positions but we *can* predict the probability of detection at each point.

Heisenburg uncertainty principle as a FUNDAMENTAL limit on our knowledge of position and momentum $\Delta x \Delta p_x \geq \hbar/2$

Quantum particles combine classical wave and particle properties. We detect photons/electrons as individual events like classical particles. Yet classically, individual particles have individual trajectories which can be completely known, but quantum particles have an irreducible uncertainty on position and momentum.