Introduction to modern physics: Special Relativity

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I'm Prof Chris Done and I get to teach you this section of the course termed 'modern physics'. I got really excited about teaching this course. It was always my favourite bit of being an L1 tutor, that part where physics goes strange! Up till now, what you've seen has been very much built with the physical intuition you get from just looking at the world around you. But all that hard won intuition is not going to help you here, where we extend into the unfamiliar world - the world of the very fast which is special relativity, and the world of the very small which is quantum mechanics. Its not that your physical intution is wrong - it works great for the slow and macroscopic. it just needs extending when we move to more extreme environments.

What I'm going to do in both sections of this course is take you through how physics works in these more extreme environments. I'll get you to build up new physical intuition by doing problems, calculating what happens. Then when you have some new feeling for how this works, I'll talk about the bigger picture, how to think like a relativist, how to think about a quantum world. But we'll get there via the maths, doing the calculations, and doing some more calculations, so you get to build up experience of these unfamiliar worlds.

Books, as ever in first year is Young and Freedman, website lecture notes as ever are on duo.

1 Special Relativity (chapter 37 in YF)

The key thing in special relativity is knowing who is seeing what. Its all about constructing a reference frame for the problem. We'll do this first in 'standard' classical mechanics - called Newtonian or Galilian, and then think about how to change this once we start to think about travelling at speeds close to the speed of light.

1.1 Reference frames in classical mechanics: YF37.1

We can set up a reference frame using a set of axes (usually cartesian). A key concept is an inertial reference frame. In this frame an observer does not experience any net forces - they are not accelerating. Inertial reference frames move at constant velocity with respect to each other.

Quantities are observed differently in different inertial frames, but absolute motion cannot be detected. They are absolutely equivalent. This is called the 'principle of relativity'. For example if two observers are in inertial frames S and S' where S' moves with velocity u relative to someone in S, then someone is S' sees S move with velocity $-u$. and we can transform between events with coordinates in S' $(x'(t'), y'(t'), z'(t'))$ to coordinates in S $(x(t), y(t), z(t))$. I've said t' for symmetry but 'of course' $t' = t$ in Newtonian Physics.

Note: prime does NOT mean derivative. It is common practice to use notation such as S and S' to denote different frames in relativity, so we will always use df/dt for derivative rather than the shorthand f'

Einstein's Principle of Relativity says that once the laws of physics have been established in one inertial frame, they can be applied without modification in any other inertial frame. Both the mathematical form of the laws of physics and the numerical values of basic physical constants that these laws contain are the same in every inertial frame. So far as concerns the laws of physics, all inertial frames are equivalent

But are they? There is already a really interesting point here. There is no absolute frame of motion for reference IN EMPTY space. But space is NOT

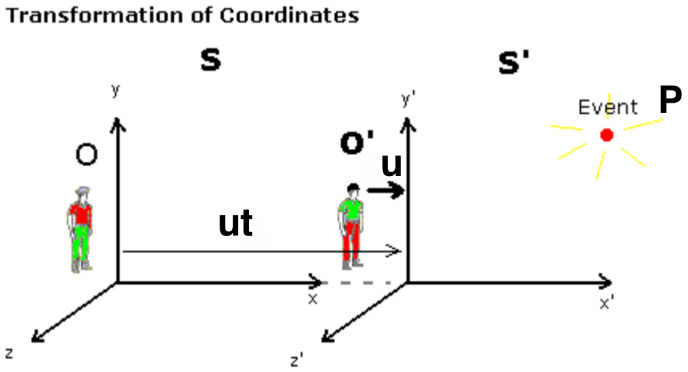

Figure 1:

empty, it is filled with cosmic microwave background radiation. So we CAN define a 'special' inertial frame, one which is at rest compared to the cosmic microwave background. And this can provide an absolute reference frame for the universe, one I can agree on with someone on a planet round a star in Andromeda. But its not special in any other way, its just the frame comoving with the Big Bang expansion.

Lets make things simple - lets have u point in the directon of the $x-x'$ axis, only do things in 2D, and make the origins coincide at $t=t'=0$. An event P in S' has initial position $x0', y0'$. If its stationary in S' then an observer, O', in S', sees this at the constant position $x'(t')=x0'$, $y'(t')=y0'$

Another observer, O in S, instead sees this moving and measures coordinates

$$x(t)=x0'+ut \quad y(t)=y0'$$

This is the Newtonian/Galilean world we are used to.

Suppose instead we'd got spacetime coordinates in S and wanted to figure them out in S'. We just replace u with $-u$ and primes with unprimes $x'=x-ut$, $y'=y$, $z'=z$ and of course $t'=t$.

We can use these Newtonian/Galilean coordinate transformations to transform velocities between inertial frames. Suppose there is an object P which is moving with velocity vx', vy' as measured by an observer in S'. Now lets work out what happens to its velocity as seen in S.

$$v_x = \frac{dx}{dt} = \frac{d(x' + ut)}{dt} = v_x' + u$$

$$v_y = \frac{dy}{dt} = \frac{dy'}{dt} = v_y'$$

We can do acceleration also

$$a_x = \frac{dv_x}{dt} = \frac{v_x' + u}{dt} = \frac{dv_x'}{dt} = a_x'$$

$$a_y = a_y'$$

Acceleration is the same even if velocity and position are not, so Newton's laws still work - $F = ma = ma'$.

Newton First law (inertia): if no external forces are acting then an object at rest will remain at rest or if its moving it will continue to move with constant velocity

Newton Second law (acceleration): when an external force acts upon an object it will accelerate in proportion to the magnitude of the net forces and in the direction of that force. The constant of proportionality is the mass so $\vec{F} = m\vec{a}$.

Inertial reference frames are ones in which Newton's first and second laws apply. It follows that general conservation principles of momentum and energy also apply in inertial frames. Any inertial frame moving with constant velocity with respect to another inertial frame is also an inertial frame.

1.2 Approaching light speed

Now lets go faster - an observer sees a spacecraft move past at +1000 m/s. The spacecraft sends out a probe whose speed is 2000 m/s relative to the spacecraft.

We set up the frames so that the spacecraft is in S', where it is at rest. The probe has velocity $vx' = 2000$ m/s in this S' frame. The spacecraft frame S' moves with velocity $u = 1000$ m/s relative to an observer O in S.

With Galilean transforms, the observer in S sees the probe as moving at $v_x = v'_x + u = 3000$ m/s.

Instead of launching a probe, the spacecraft turns on a searchlight which travels at speed c relative to the spacecraft. Classical mechanics says the observer in S should see this at speed $v_x = c + u > c$. Maxwells equations says it travels at c.

So something doesn't work. Either there is such a thing as a special inertial reference frame in which Maxwells equations work. Perhaps space is filled with something - an aether - in which electromagnetic waves propagate? Or maybe velocities don't add up the way they should in classical physics....

One way to test this is to look. The Michelson-Morely experiment took the idea of an aether and figured out that the motion of the Earth means that an experiment would move with respect to this aether. so we could detect motion relative to the frame of the aether by looking at the speed of light. This would mean that the laws of physics were NOT the same in all inertial frames, but that there was a special frame for electromagnetism, the frame of the aether.

They measured this with an interferometer, with axes at 90 degrees. The Earth moving with respect to the aether means that they should see a shift in the interference pattern due to the changing speed of light when they rotated it with respect to the aether. They didn't.

There is no aether/special inertial frame for light. Maxwell is right, its classical physics which is wrong.