1.3 Postulates of special relativity YF37.2-37.3

we saw last lecture that there is no aether/special inertial frame for light. Maxwell is right, light ALWAYS travels at speed c no matter what frame we are in. Its classical physics which is wrong. velocities close to the speed of light DO NOT ADD in the 'standard' Galiean way.

Einstein's reaction to the failure to detect an 'aether' was radical. He embraced it wholeheartedly!

Einstein 1: the laws of physics are the same in every inertial frame of reference (this is in the spirit of Newtonian physics, as we saw from last lecture)

Einstein 2: the speed of light in a vacuum is the same in all inertial frames of reference and is independent of the motion of the source (this takes Maxwells equations at their word! so in some sense we didn't need to say it as its just a specific case of a law of physics in the 1st postulate - but its good just to be explicit because the implications are profound.)

We saw last lecture that the speed of light being constant means that the 'obvious' way in classical mechanics to transform velocties between inertial frames is not correct. Velocity is distance/time so we have to change our definitions of either distance or time or both (its both!)

1.4 light as the ultimate speed limit

The first thing that this tells us is that the speed of light is an ultimate speed limit, that no inertial observer can travel at c. we can see this most easily by doing a thought experiment. Suppose a spacecraft at rest in S' moves with velocity $u = c$ relative to S, and then turns its headlights on. In S, the headlight must travel at c. but so does the spacecraft, so the light is always in the same place as the spacecraft. Yet in the spacecraft in S' they have to see it move away from the spacecraft at c, so they cannot be at the same point in spacetime. As a 16 year old Einstein wondered what he would see if he were travelling at c - but the answer is you can't.

1.5 Simulteneity

A more subtle and disturbing effect of the speed of light being constant in all frames is that events that are simultaneous for one observer need not be simultaneous for another.

Suppose we have multiple lightning strikes as in example YF37.5. Lightning strikes points A' and B' at the back and front of a train carriage and they also burn the ground at A and B. There are two observers, one on the ground, equidistant between A and B and one in the train, equidistant between A' and B'.

These strikes happen simultaneously simultaneously as seen from the observer outside the train.

But the observer in the train is travelling forward. so sees the light from B' before the light from A'. But they know they are equidistant between A' and B', and that light always travels at c. In which case, if A' arrives first then it must have happened first.

Two observers do not need to agree on the order of events which occur at different positions.

A key point here is that SIMULTENEITY is not the same as CAUSALITY. The lightning strike at the front does not cause the lighting strike at the back, so the order in which they happen is not really important. its just a shift in viewpoint.

Its like whether I see a firework explode exactly along my line of sight to the top of the eifel tower - if I was standing somewhere different I'd see it explode to the right or left of the tower. Only if the firework was ATTACHED to the tower would every observer position agrees that it explodes exactly in line with the tower.

whether two events separated in space are simultaneous depends on the motion of the observer. Simulteneity is not an absolute concept (though causality is)

But this means that the time interval between two events is different in

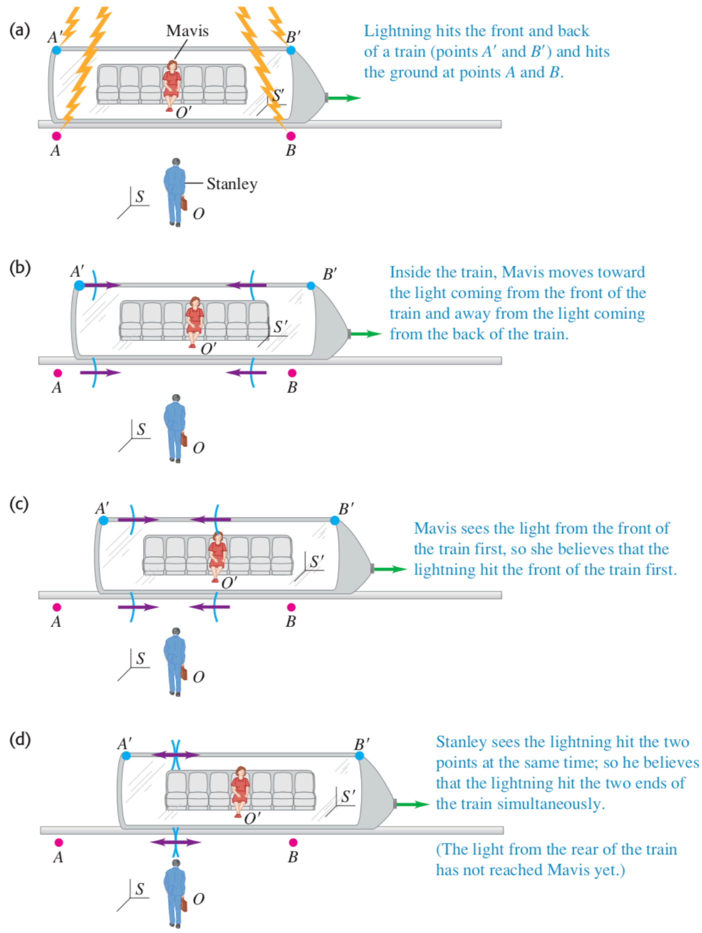
(a)
A′
Mavis
B′
Lightning hits the front and back of a train (points A′ and B′) and hits the ground at points A and B.
S′
O′
A
B
Stanley
S
O
(b)
A′
B′
Inside the train, Mavis moves toward the light coming from the front of the train and away from the light coming from the back of the train.
S′
O′
A
B
S
O
(c)
A′
B′
Mavis sees the light from the front of the train first, so she believes that the lightning hit the front of the train first.
S′
O′
A
B
S
O
(d)
A′
B′
Stanley sees the lightning hit the two points at the same time; so he believes that the lightning hit the two ends of the train simultaneously.
S′
O′
A
B
(The light from the rear of the train has not reached Mavis yet.)
S
O

Figure 2:

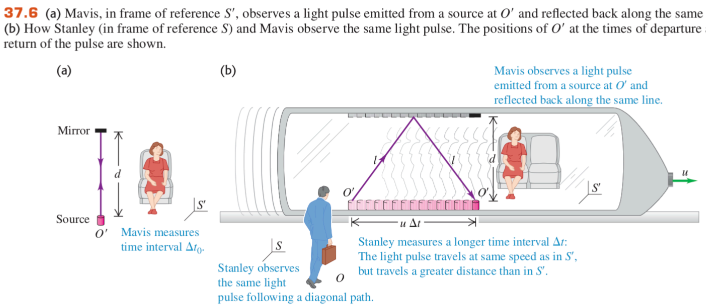

Figure 3:

different frames.

1.6 Relativity of time intervals - time dilation

We have observers in frame S, and S'. S' moves with velocity u relative to S, along the x-x' direction and their axes O and O' align at t=t'=0.

In S' a beam of light goes vertically upwards from the origin O' and reflects back vertically downwards from a mirror after vertical distance d as measured in S'. In S' the time this takes is $\Delta t' = 2d/c$

In S, the light path is not vertical, but moves horizontally by $u\Delta t$ as well as vertically. so then the path length up to the mirror is ℓ where $\ell^2 = d^2 + (u\Delta t/2)^2$, and the total path length is 2ℓ.

Because light always travels at c this means the time interval measured by someone outside the carriage for the light to get back to the emitter is

$$\Delta t = 2\ell/c = \frac{2}{c}\sqrt{d^2 + \left(\frac{u\Delta t}{2}\right)^2}$$

square this and get

$$\Delta t^2 = \frac{4}{c^2}\left[d^2 + \frac{u^2 \Delta t^2}{4}\right] = \frac{4d^2}{c^2} + \frac{u^2}{c^2}\Delta t^2$$

but $\Delta t' = 2d/c$ and $\beta = u/c$ then $\Delta t^2 = (\Delta t')^2 + \beta^2 \Delta t^2$ and hence

$$\Delta t = \frac{\Delta t'}{\sqrt{1-\beta^2}} = \frac{\Delta t'}{\sqrt{1-u^2/c^2}}$$

since $(u/c)^2 < 1$ then $\Delta t > \Delta t'$. The time for the light pulse to go up and down is longer in S than in S'.

Time intervals are measured by clocks. So 'ticks' on a clock as measured in a frame in which the clock is at rest are shorter than those measured in a frame which moves relative to the clock. If you see the clock move, then a clock in your frame measures slower time intervals than the clock in frame moving relative to you. Fast clocks run slow.

The size of this effect is very small in normal life. e.g. $d = 1.5$m and $u = 300$m/s (a plane) then $\Delta t' = 2d/c = 10^{-8}$s inside the plane, whereas someone on the ground measures $\Delta t = \gamma(u)\Delta t'$

$u/c = \beta \ll 1$ so we can use the approximation $(1+x)^n = 1 + nx + \ldots$ where $\gamma = (1-\beta^2)^{-1/2}$ so $x = -\beta^2$ and $n = -1/2$ so $\gamma \approx 1 + (-\beta^2)(-1/2) = 1 + \beta^2/2 = 1 + (300/3 \times 10^8)^2/2 = 1 + 10^{-12}/2$. Thus $\Delta t = (1 + 10^{-12}/2)\Delta t'$ and so the fractional difference in times measured is tiny at

$$(\Delta t - \Delta t')/\Delta t' = 10^{-12}/2$$

But now take $u = 0.99c$ then $\gamma = 7.1$ and $\Delta t = 7.1\Delta t'$ and the fractional change is 6.1, or for $u = 0.9999c$, where $\gamma = 70.7$ the fractional change is ~ 70. $\gamma(u)$ asymptotes to ∞ as $u \to c$.

1.6.1 Proper time

There is only one inertial frame in which we are in the same frame as an event, but infinitely many which are moving relative to it. So time intervals

37.8 The quantity $\gamma = 1/\sqrt{1 - u^2/c^2}$ as a function of the relative speed u of two frames of reference.

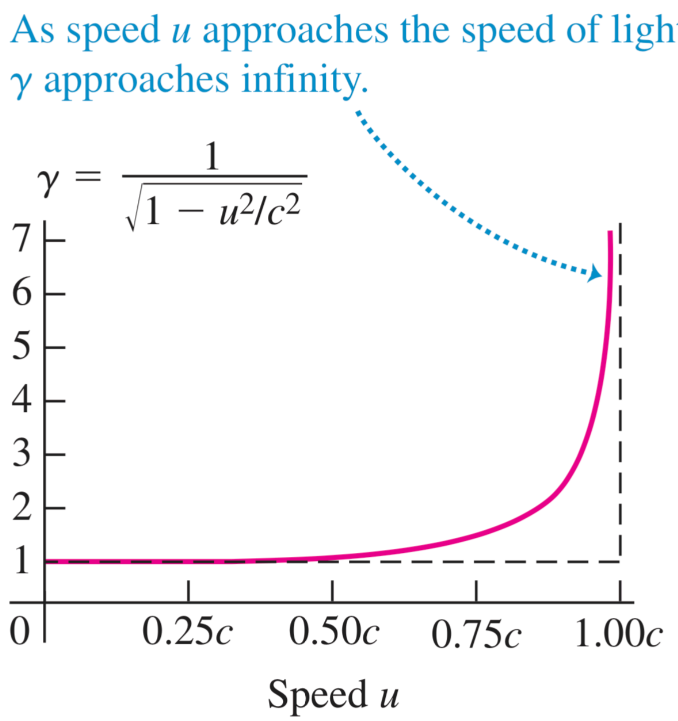

Figure 4:

measured in the same frame as the event have a more fundamental quality than those in any other frame. We use the term **proper time**, T_0, to describe the time interval between 2 events which occur at the same point.

(CAUTION: in the standard setup of S and S', proper time is measured in S' - so you might expect it to be primed, but by convention it does not)

Proper time is always the shortest, all other frames, S, see S' move with velocity u so they measure time intervals which are longer by a factor $\gamma(u)$

1.6.2 Example

e.g. example YF37.1 high energy particles from space interact with atoms in the earths upper atmosphere to produce muons. These decay in their rest frame with lifetime $\Delta t' = 2.2 \times 10^{-6}$ s.

if the muon is moving with respect to the Earth with $u = 0.99c$ then what is its mean lifetime as measured on earth?

$\gamma = 1/\sqrt{1-\beta^2} = (1-0.99^2)^{-1/2} = 7.09$ so $\Delta t = 7.09 \times \Delta t' = 1.56 \times 10^{-5}$ s.

Their half life is $7x$ longer as seen on Earth (a frame in which the muon moves with speed $u = 0.99c$) than experienced in the rest frame of the muon.