1.6.3 Example

Things can get tricky. but generally only because they are described in a confusing way.

Example YF 37.3. A spacecraft with traveller (Mavis) zips past an observer (Stanley) on Earth at $u = 0.6c$. As they pass they both set their clocks to zero. A short time later stanley measures mavis passes a spacestation at a distance stanley measures to be 9×10^7 m away.

What does stanley;s timer read as mavis passes the space station

what does Mavis's timer read as they pass the space station

The standard axes setup is to put the spacecraft observer/mavis/O' in frame S' and the earth observer/stanley/O in frame S. Set $t' = t = 0 = x = x'$ when the observers pass ie when $O' = O$

Stanley measures distance of 9×10^7 m away and a speed of $u = 0.6c$ so this takes $\Delta t = \ell/u = 9 \times 10^7/(0.6 \times 3 \times 10^8) = 0.5\ s$

In S' these events happen at the same point in space which is their own position - first they line up with O and then they line up with the space station. So O' (mavis) is at rest so measures the proper time, which is always the SHORTEST possible time interval. so we need $\Delta t' = \Delta t/\gamma = 0.4$ s

Now it gets more confusing. Stanley blinks as Mavis flys past. Mavis measures the blink to take 0.4 s. How long does Stanley think this takes?

its very tempting to say 0.5 s but this is NOT correct because its a DIFFERENT pair of events. Now its STANLEY who is at rest with respect to the two events (time taken for Stanley's eyelid to go down and up).

So we should relabel the frames. STANLEY is now in the frame S' in which the event is at rest. and MAVIS is in S which is moving relative to S' with velocity $-u$. but time dilation depends only on u^2 so direction doesn't matter. so then STANLEY is in S' so $\Delta t' = \Delta t/\gamma = 0.4/1.25 = 0.32$ s.

This gets us nicely back to our original discussion of simulteneity. Mavis passes the space station after 0.4 s, and sees Stanley finishing his blink. Yet to Stanley, he thinks his blink ends after 0.32 s, way before mavis passes the space station at 0.5 s.

But these events are not causally related so the order in which we see them is not important.

1.6.4 Twin paradox

But surely we can make this causal. Take twins eartha and astrid. Astrid flys away from earth at high speed u, so all time intervals in her rest frame are shortest - including heartbeats! so she ages more slowly than eartha.

But since sign doesn't matter she can turn around and come back at the same speed and still age less. but then she can meet up with her twin and then Astrid is younger than Eartha.

But all inertial frames are relative so surely Eartha could say the same - that she has gone away from astrid at $-u$ and then turned around and met up with her. so Eartha should be younger!

Who is correct? when they meet they should be able to tell who is actually younger!

In fact its astrid. The key is that these frames are not symmetric. Astrid has swapped inertial frames whereas Eartha has always been in the same one (its easier to think of it this way than to get tangled with acceleration/deceleration). Astrid is indeed younger, though its clearer to see this by folding in what is happening to space as well as to time.

1.7 Relativity of length

Speed=distance/time. and we've seen that time intervals have distorted because of the fixed speed of light. Now we will see that length does as well.

We measure can length of an object like a car by making marks on a stationary (relative to the car) pavement at the front and back of the car and measuring between them.

If instead the car is moving with respect to the road then we have to make the marks simultaneously to get the true length of the car - if instead we marked the position of the back of the car a bit later than when we measured the front hen we could get negative length as the back is now further forward than the front was when we measured it.

But then length involves marking simultenously the front and back - so we are back into the issue we had where simultaneous is a frame dependent concept (though causality isn't).

1.8 Lengths parallel to motion

Set up our standard frames S and S', so S' is moving with velocity u wrt S. Put a ruler in S', and measure its length in this rest frame as ℓ'. Attach a light to one end, and a mirror to the other. The total distance is go along the ruler and back to the same point is $2\ell'$ in the rest frame of the ruler, and the time interval (proper time as its all in the rest frame of the ruler) between the light signal starting and being recieved is $\Delta t' = \Delta t'_1 + \Delta t'_2$. In the rest frame, $\Delta t'_1 = \Delta t'_2$ as the light trip is symmetric there and back so $c = 2\ell'/\Delta t'$

In frame S we know that the total length that the light has to travel is the length of the ruler in this frame, which is ℓ plus the frame shift. The length to the mirror is then $\ell_1 = \ell + u\Delta t_1$ and it goes at the speed of light so $c = \ell_1/\Delta t_1$ so $c\Delta t_1 = \ell + u\Delta t_1$ and $c\Delta t_1 = \ell/(c-u)$

On the way back we have $\ell_2 = \ell - u\Delta t_2$ and $c = \ell_2/\Delta t_2$ so $c\Delta t_2 = \ell - u\Delta t_2$ so $c\Delta t_2 = \ell/(c+u)$

The total time measured in S is $\Delta t = \Delta t_1 + \Delta t_2$

$$\Delta t = \frac{\ell}{c-u} + \frac{\ell}{c+u} = \frac{2\ell/c}{1-u^2/c^2} = \gamma^2 2\ell/c$$

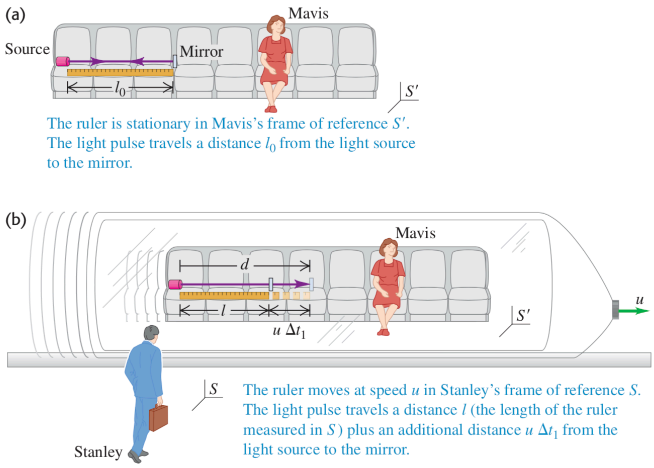

Figure 5:

We also know how the time intervals change from one frame to another $\Delta t = \gamma \Delta t'$

Substitute in and get $\gamma \Delta t' = \gamma^2 2\ell/c$ so $\Delta t' = \gamma 2\ell/c$

In the rest frame we had that $\Delta t' = 2\ell'/c$ so we equate and so $2\ell'/c = \gamma 2\ell/c$ which gives us $\ell = \ell'/\gamma$.

Since $\gamma(u) \geq 1$ then the lengths measured in the moving frame are smaller than lengths measured in the rest frame. This is length contraction. It is real, not an optical illusion, in the same way that time dilation is real - we really age less quickly if we move fast - and we really take up less space if we move fast.

When $u \ll c$ then $\ell \sim \ell'$ and we are back to classical mechanics. However, when $u \to c$ then $\ell \ll \ell'$

We call ℓ' - length measured in the rest frame of the - PROPER DISTANCE ℓ_0 (again beware the lack of primes but it is measured in the primed frame) - in the same way that time intervals measured at a single point are PROPER TIME

Example FY37.4 A spacecraft flies past the earth at a speed of 0.99c. A crew member on board the spacecraft measues its length to be 400m. What length do observers on earth measure?

S' is the frame of the spacecraft. $\ell' = \ell_0 = 400$ m is proper length.

S is earth frame, so $\ell = \ell'/\gamma$ and $\gamma = 1/\sqrt{1-0.99^2} = 7.09$ so $\ell = 400/7.09 =$ 56.4 m

Example YF37.5. (continuing from above) Suppose 2 observers on Earth are 56.4 m apart. How far apart does the spacecraft crew measure them to be?

This is a DIFFERENT EVENT so change frames. Now the EARTH is the rest frame so call it S', and the spacecraft is the new frame, S, that we want to consider. They still have relative velocity u so $gamma = 7.09$ again. But now we are looking at distance in S which is the SPACECRAFT frame relating to a proper distance ℓ' measured in a rest frame so $\ell = \ell'/\gamma = 56.4/7.09$ so $\ell = 7.96$ m

THIS IS NOT THE PROPER LENGTH OF THE SPACECRAFT. As measured on earth it is the length of the spacecraft when the nose and tail are simultaneously measured. In the spacecraft frame these two positions are only 7.96m apart and the nose is 400m in front of the tail. The nose passes O2 before the tail passes O1.

1.8.1 lengths perpendicular to motion

Actually we have already assumed that this didn't change in our discussion of the time transformations! But this should be as at $t = t' = 0$ then the base of the ruler coincided with the x-x' origin. but it lies directly on the y-y' axis. so this lines up at $t = t' = 0$.