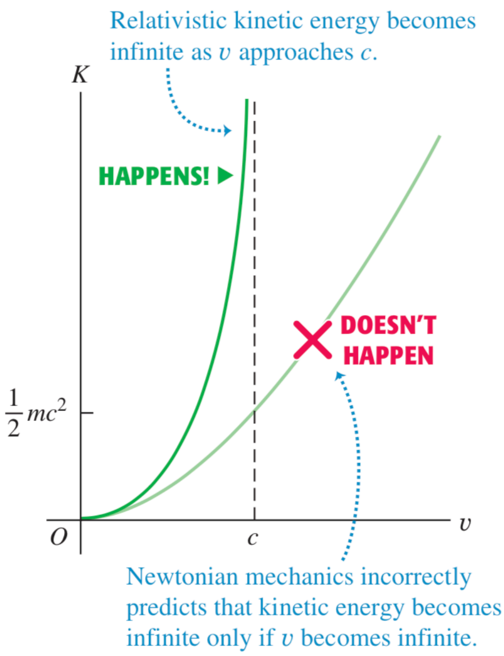

Figure 10:

2.4 Relation between momentum and energy

In the same way that classical KE and momentum can be related via $KE = p^2/2m$ we can particle energy and momentum in relativisitic mechanics as

$E = \gamma mc^2$ and $p = \gamma mv$

re-write these as $E/mc^2 = \gamma = (1-\beta^2)^{-1/2}$ so $(E/mc^2)^2 = 1/(1-\beta^2)$ while $p/mc = \gamma\beta$ so $(p/mc)^2 = \gamma^2\beta^2$

$$\left(\frac{E}{mc^2}\right)^2 - \left(\frac{p}{mc}\right)^2 = \frac{1}{1-\beta^2} - \frac{\beta^2}{1-\beta^2} = \frac{1-\beta^2}{1-\beta^2} = 1$$

$E^2 - p^2m^2c^4/(m^2c^2) = m^2c^4$

$E^2 - p^2c^2 = m^2c^4$ or $E^2 = (pc)^2 + (mc^2)^2$

This implies that a particle with no rest mass can still have energy and momentum - these are photons where $E = pc$.

2.5 Rest mass energy

$E = K + mc^2 = \gamma mc^2$ so even if the particle is at rest it has energy $E = mc^2$ this is energy associated with rest mass rather than with energy of motion

There is clear experiemntal evidence for this

example: Nuclear fusion

$$\frac{1}{1}1H + \frac{7}{3}Li \rightarrow \frac{4}{2}2He + \frac{4}{2}He$$

these have atomic masses of $^1_1H = 1.007825$, $^7_3Li = 7.016005$ so total before is 8.023830u

$^4_2He = 4.002603$ so total after is 8.005206.

We LOST some mass. so we must have GAINED some energy. The energy gained (to KE or the particles) is a net heat of $\Delta Q = 0.018624uc^2$ where $u \approx m_p$.

total energy is $\sim 8uc^2$ so gain in energy is $\Delta Q/Q = 0.018624/8 = 0.002$ i.e. 0.2% of the rest mass energy can be converted to kinetic energy in this reaction. This doesn't sound a lot, but it is actually! Its not far from the most efficience fusion process, which is

$$\frac{1}{1}H + \frac{1}{1}H + \frac{1}{1}H + \frac{1}{1}H \rightarrow \frac{4}{2}He$$

mass before $4 \times 1.007825 =$, afterwards 4.002603. so $\Delta Q = 0.0286uc^2$ and $\Delta Q/Q = 0.007$

This is the reaction which powers the Sun and (most of) the stars we see in the night sky.

MASS NEED NOT BE CONSERVED IN COLLISIONS. The thing we care about is total energy.

example: Neutral pions are unstable particles. they decay to PHOTONS. $\pi^0 \rightarrow \gamma + \gamma$

What is the energy of the photons in the rest frame of the pion?

initial $E_\pi = mc^2$ and $p_\pi = 0$ as it is at rest

final $E = E1 + E2 = h\nu_1 + h\nu_2$ and $|p1| = E1/c$ and $|p2| = E2/c$.

conserve momentum and we need $p1 = -p2$ so $E1 = E2$ - there are two identical photons going in opposite directions,

conserve energy $E = 2h\nu$ and so $2h\nu = m_\pi c^2$

2.6 relativisic collisions and kinematics

We are going to limit ourselves to 1D motion for particles as we've only really done relativity in 1D. we are going to remember that we don't need to conserve mass.

Example: π^0 production: YF example 37.11

Two protons, each with mass 1.67×10^{-27} kg are initially moving in opposite directions. They continue to exist after a head on collisions that produces a neutral pion of mass 2.4×10^{-28} kg. If all particles are at rest after the collision, what is the initial speed of the protons.

momentum before is zero as the particles have same speed but opposite direction.

total energy is conserved. each initial proton has rest energy and kinetic so $E1 = E2 = \gamma(u) m_p c^2$

all particles are at rest afterwards, so only rest mass energy.

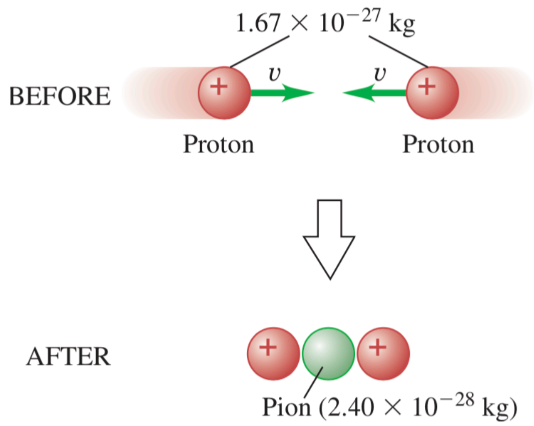

Figure 11: Pion production

$2\gamma_p(u)m_pc^2 = 2m_pc^2 + m_\pi c^2$

$\gamma(u) = (2m_p + m_\pi)/(2m_p) = 1 + m_\pi/(2m_p) = 1.072$

$\gamma = (1-\beta^2)^{-1/2}$ so $\beta^2 = (\gamma^2-1)/\gamma^2$ and $\beta = 0.36$

Example - kaon production! (its a problem in YF37, number is 69 in my version.

some of the incident KE is used to create rest mass energy of the new particles so this is NOT an elastic collision! and it won't conserve mass either!

$$p + p \to p + p + K^+ + K^-$$

The rest energy of each Kaon is 493.7MeV, the proton is 938.3MeV.

Calculate the minimum kinetic energy for proton 1 which allows this to occur if proton 2 is initially at rest.

This is much easier to do in the centre of mass frame as here the 2 protons have equal and opposite (unknown) velocities of u and $-u$ and EVERYTHING is at rest afterwards.

Conserve energy: before collision $\gamma(u)m_pc^2 + \gamma(-u)m_pc^2 = 2\gamma(u)m_pc^2$

After collision, everything is at rest for a minimum energy collision so $2m_pc^2+$

$2m_K c^2$.

Equate and solve for $\gamma(u) = \gamma(-u)$ and get

$\gamma(u) = 1 + m_K/m_p = 1 + 493.7/938.3 = 1.53$ so $1 - \beta^2 = 1/1.53^2$ and $\beta = 0.76$.

Now we need to transform this to a velocity on one of the protons, so its at rest with respect to the other one.

Pick the one moving to the right, make this S' so we have our standard setup for $u = +0.76c$. in our central frame, we had $v_x = 0.76c$ for the particle on the left, and $v_x = -0.76c$ for the particle on the right.

transform to the primed frame, so we see what the stationary particle sees

particle on the left:

$$v'_x = \frac{v_x - u}{1 - v_x u/c^2} = 0$$

this is as expected as we wanted to be in the rest frame of the particles on the left!

particle on the right - this is what we want!

$$v'_x = \frac{v_x - u}{1 - v_x u/c^2} = -0.76c - 0.76c/(1 - (-0.76c)(0.76c)/c^2 = -1.51/1 + 0.57 = -0.96c$$

so the particle on the right has very high velocity in the rest frame of the particle on the left. its kinetic energy $(\gamma - 1)m_p c^2 = (3.68 - 1)m_p c^2 = 2.68.3938 MeV c^2 = 2515 MeV$ This is much more than the rest mass energy needed to produce the kaons, as in this frame the second proton is moving after the collision.