3 How to think about relativity

Most of this material is now in the new section of FoP1, on Collisions, Conservation and Fields. But I've put it here explicitly as there are some very nice results which also help us understand what special relativity is (and is NOT). its NOT that everything is relative. The order of unconnected events is relative (simultaneous for one observer does NOT mean simultaneous for another), time intervals and lengths depend on how you are moving.... but there are SOME things which are NOT relative. These are conserved quantities - they are invariant and do NOT depend on the frame/motion of the observer.

3.1 Collisions

We saw that for a single particle $E^2 - (pc)^2 = m^2c^4$. and if all we are considering is a single particle then whatever frame we transfer to, m^2c^4 will be the same. so suppose its in a frame where it has velocity βc. then

$$E^2 - (pc)^2 = \gamma^2 m^2 c^4 - \gamma^2 m^2 v^2 c^2 = \gamma^2 m^2 c^4 (1 - v^2/c^2) = m^2 c^4$$

irrespective of the value of β.

But when we have multiple particles involved, what happens next?

In the Relativity homework (Q3, self assessed) you did 2 identical particles, in the centre of momentum frame they had $\beta = \pm 0.41$ ($\gamma = 1.10$) and in the rest frame of one, the other $\beta = 0.7$ ($\gamma = 1.40$).

Centre of momentum frame total energy $E_{tot} = 2\gamma_{CM} mc^2 = 2.2mc^2$ and total momentum is $p_{tot} = 0$ (CM frame!) so $E_{tot}^2 - (p_{tot}c)^2 = 2.2^2 m^2 c^4 = 4.81 m^2 c^4$. This is NOT the sum of the particle masses squared, as thats $4m^2c^4$. But it is still a conserved quantity as you can see from switching to the rest frame of one of the protons, where $E_{tot} = \gamma_E mc^2 + mc^2 = 2.40mc^2$ and $p_{tot} = \gamma m \beta c + 0 = 0.98mc$ but $E_{tot}^2 - (pc)^2 = (2.40^2 + 0.98^2) m^2 c^4 = 4.80 m^2 c^4$ as before!

so this IS an invariant but its NOT the sum of the masses. Its the total energy (rest plus kinetic plus electromagnetic) in the centre of momentum frame.

its obvious that its not mass when we start to think about a neutral pion, decaying into two photons!! the photons are massless, the pion is not.

and the final relativity homework question (Q4) makes this very explicit.

3.2 Invariant interval

There is another, even more fundamental invariant in relativity, which comes from space and time. Its not that everything is relative - there is a sneaky combination which IS absolute, and does not depend on the frame in which they are measured.

Take the Lorentz transformations, and go to the limit of small time and space intervals

$$dx' = \gamma(dx - udt) \quad dy' = dy \quad dz' = dz \quad dt' = \gamma(dt - dxu/c^2)$$

Have a look at $c^2(dt')^2 - (dx')^2 - (dy')^2 - (dz')^2$

$= c^2\gamma^2(dt - dxu/c^2)^2 - \gamma^2(dx - udt)^2 - dy^2 - dz^2$

$= c^2\gamma^2(dt^2 - 2u/c^2 dxdt + u^2/c^4 dx^2) - \gamma^2(dx^2 - 2udxdt + u^2dt^2) - dy^2 - dz^2$

$= c^2dt^2 - dx^2 - dy^2 - dz^2$

So there is some interval which is a combination of time and space which is the same in any frame. what is this interval if we were in the rest frame?

Rest frame is S'. if measuring time between two events at the same place then $dx' = dy' = dz' = 0$ and this interval is $c^2(dt')^2$ where $dt' = T_0 = d\tau$ is proper time!

so we could say $c^2d\tau^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$

in the rest frame, we also have proper length so if we measure the ends simultaneously we'd get $(dx')^2 + (dy')^2 + (dz')^2 = L_0^2 = ds^2$

so $ds^2 = c^2 d\tau^2 = c^2(dt')^2 - (dx')^2 - (dy')^2 - (dz')^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$

where ds is SPACE-TIME interval. It is the same in any frame!

so pull it all together and we have $c^2 d\tau^2 = ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 = c^2(dt')^2 - (dx')^2 - (dy')^2 - (dz')^2$ as an invariant

This is called a METRIC it shows how we use coordinates in space and time to determine the distance between any two points. This particular metric, the metric of special relativity, is called the Minkowski metric.

One way to read this is that we have a fixed speed - the speed of light - through SPACETIME. the more of that speed we put through SPACE, the less there is left to travel through TIME, so we travel through time more slowly - we age less.

if you are interested in understanding more try 'The elegant Universe' by Brian Greene

and actually, I learnt some useful stuff from the childrens book series by Russell Stannard, 'The space and time of Uncle Albert' and 'Uncle Albert and the black holes'

One of the amazing insights I got from this was WHY you can't go past the speed of light! we have just done $E = mc^2$ which says mass and energy are interchangeable. so if we have a particle and give it some kinetic energy then in some sense we have added to its inertial mass... and when the kinetic energy starts to dominate over the rest mass then we are in trouble. we accelerate it, which increases its KE, which makes its mass increase, so its harder to accelerate... it gets a bit nasty, as velocity has a direction whereas mass doesn;t so they don't have quite the same properties. but its a helpful way to see that whatever the factor is which scales between mass and energy, then this will be the speed limit.