Theoretical Astrophysics: Exercise 1

1) The position vector of any point on a cone of constant opening angle θ_0 can be written as

$$\underline{r} = r\sin\theta_0\cos\phi\mathbf{i} + r\sin\theta_0\sin\phi\mathbf{j} + r\cos\theta_0\mathbf{k}$$

a) write down the basis vectors $\underline{e}_A = \partial\underline{r}/\partial x^A$ for this surface

b) use these basis vectors to calculate the covarient metric components $g_{AB} = \underline{e}_A.\underline{e}_B$. Hence show that the line element is $ds^2 = dr^2 + r^2\sin^2\theta 0 d\phi^2$

c) state the general relation between the covariant metric components g_{AB} and contravariant metric components g^{BC}. How does this simplify for a diagonal metric? Use this to derive g^{BC}

d) Find the dual basis vectors $\underline{e}^A$ by raising the index (using the contravarient metric tensor) of the original basis vectors $\underline{e}_A$
e) Compare these with the direct calculation of $e^A = \underline{\nabla} x^A$ (they are the same modulo a normalisation factor as we don't in general normalise basis vectors). To do this, invert the coordinate transformations to find an expression for r and ϕ in terms of x, y, z [HINT $d\tan^{-1}u/du = 1/(u^2+1)$]