General relativity

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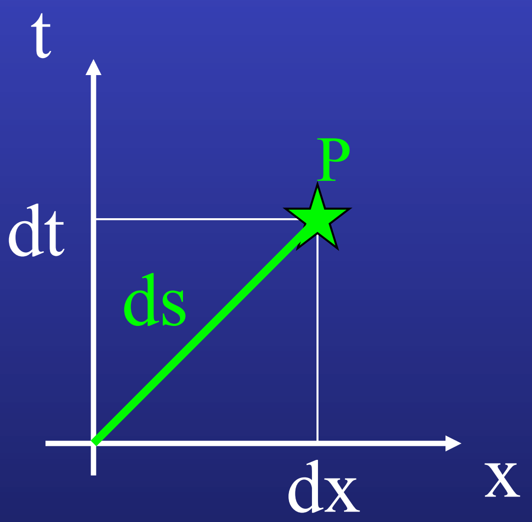
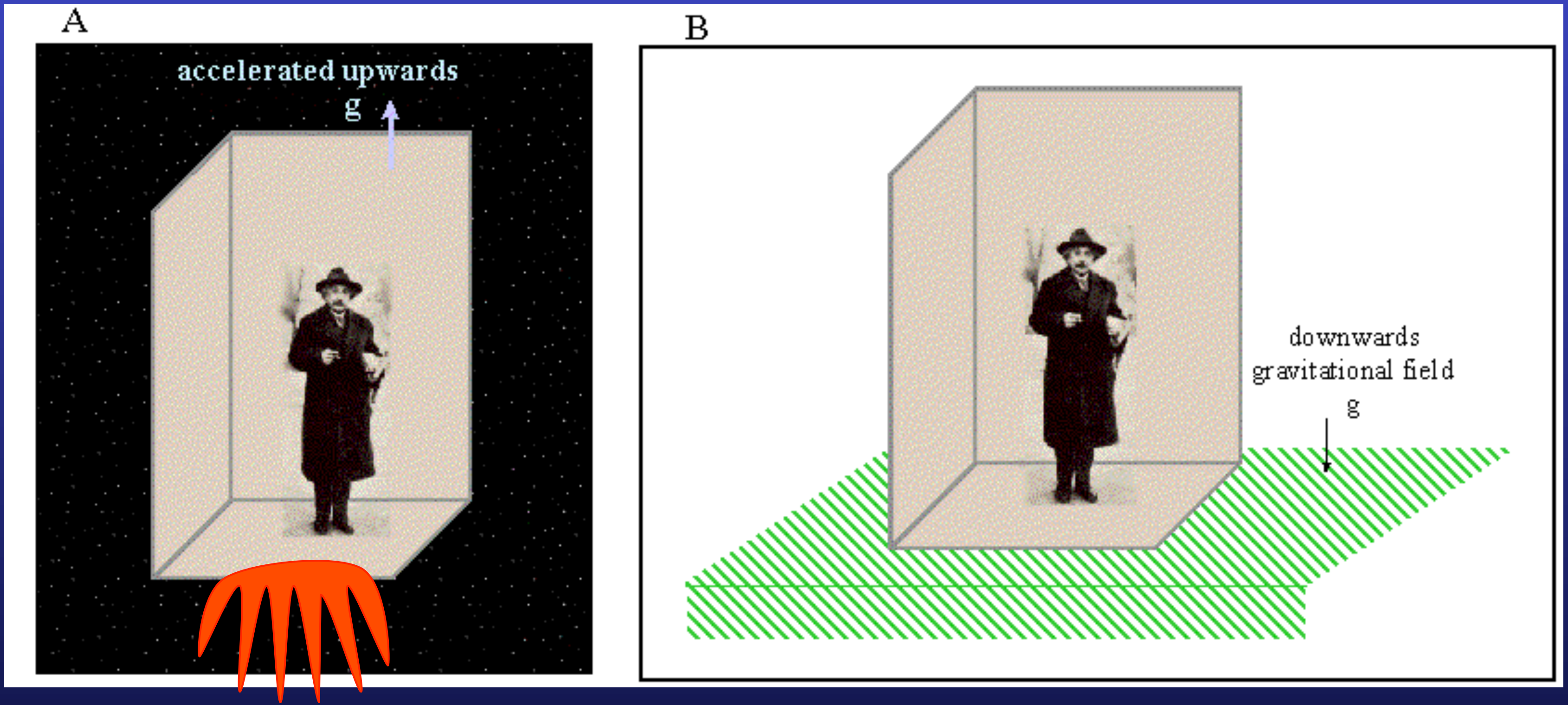
Special relativity

- Towering achievement
- Throw away ideas about fixed space and fixed time!!!
- NOT that everything is relative!
- Fixed spacetime interval
- 1D + time,

  $ds^2=c^2d\tau^2=c^2dt^2 - dx^2$
- BUT only does inertial frames. Can't handle acceleration…..



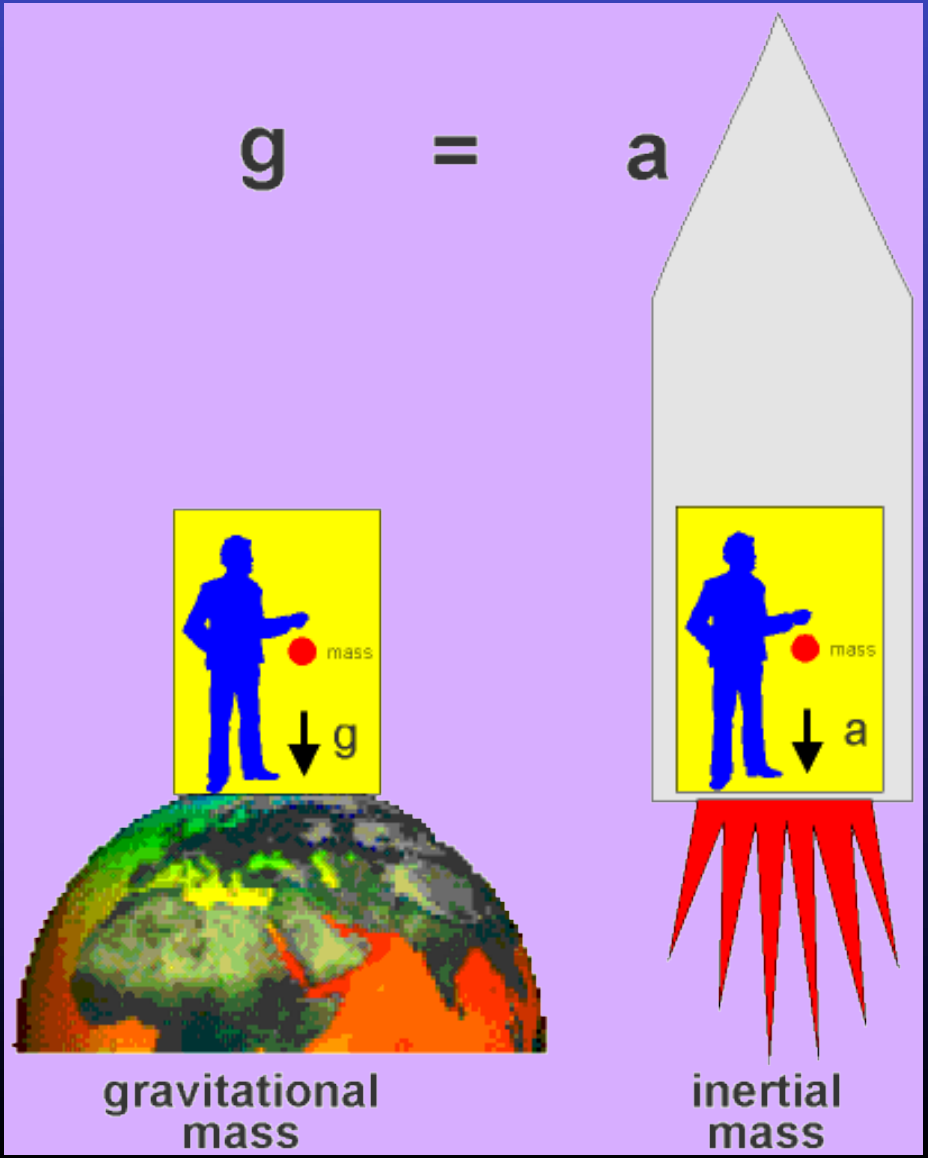
Gravity = acceleration



- difference between gravity and acceleration ?
- Look the same, behave the same…
- Maybe they ARE the same - 'happiest thought'
- Principle of equivalence: acceleration=gravity

Gravity = acceleration

- Also solves deep problem
- Inertial mass – response to accelerating force $F_i=m_i a$
- Response to gravitational force governed by 'gravitational charge' $F_g=m_g GM/r^2$
- for matter falling under gravity $F_g/F_i \propto m_g/m_i=\text{const}$
- No other force constant behaves like this eg EM $F_{em}= q\,Q/4\pi\varepsilon_0 r^2$
- $F_{em}/F_i \propto q/m_i$ different e^- p^+
- But obviously F_g/F_i same if gravity = acceleration



Acceleration: special relativity

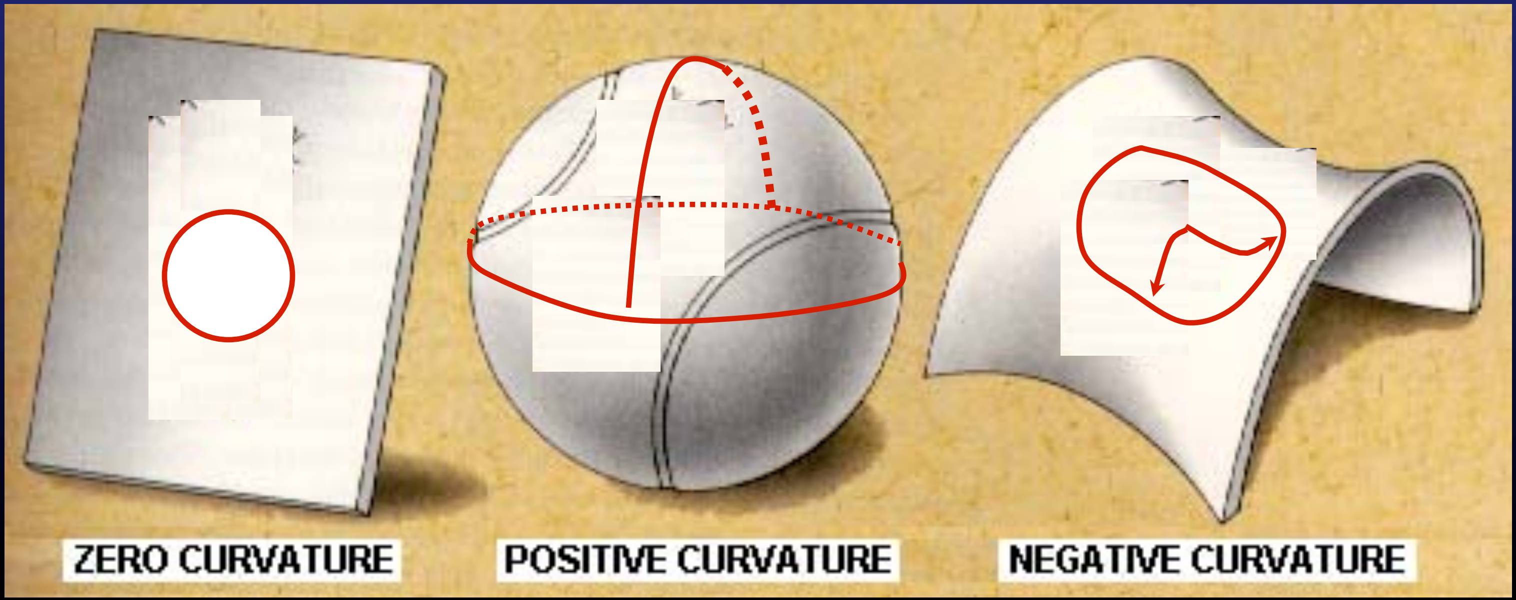
- Circular motion easiest to think about
- Measure roundabout circumference (CL) and radius (rL) by crawling around with ruler of length L
- Get ratio C/r=2π
- Now rotate ….

Acceleration: special relativity

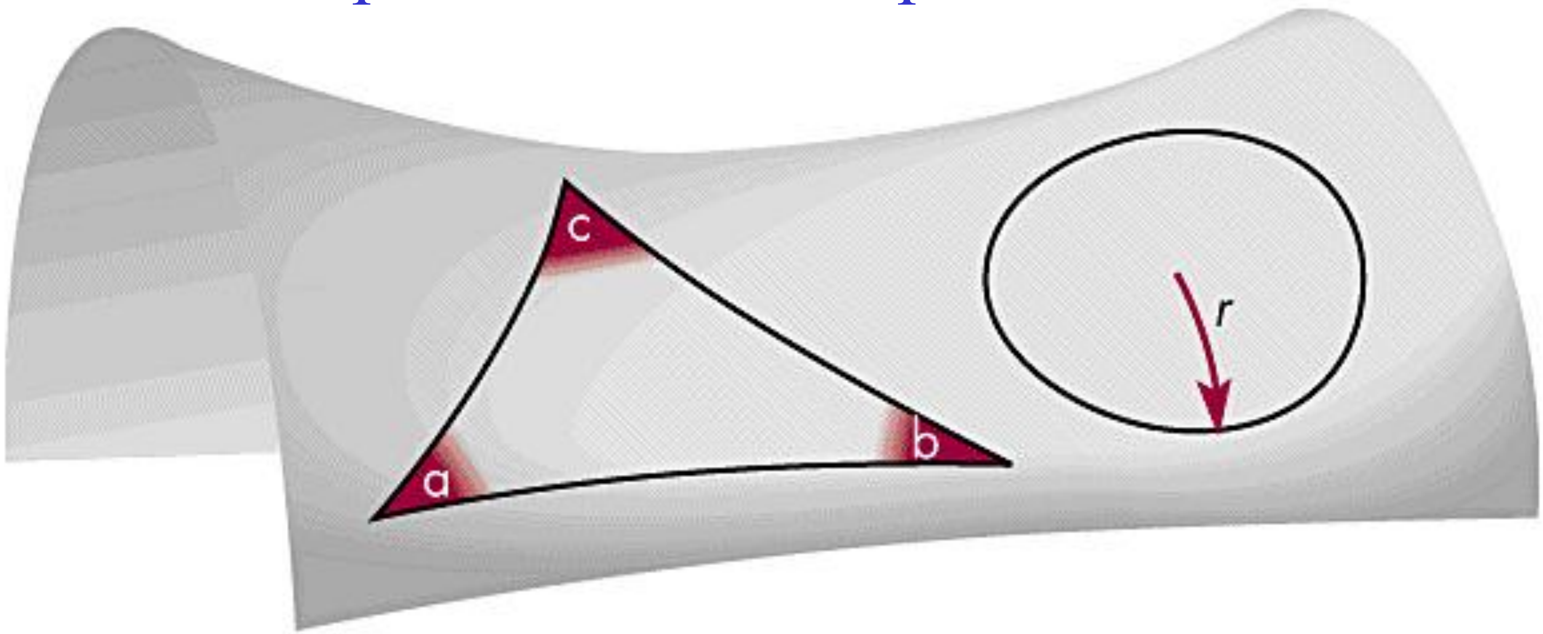
- Length contracts along direction of motion so need more ruler lengths to go round c' > c!! But radius unaffected
- Ratio c'/r > 2π
- Can't happen!! …in flat space

Curved spaces

- Can happen in curved spaces!!
- eg sphere. Circle round equator. Circumference is $2\pi r$, diameter is πr so ratio is $2 < \pi$!!!
- Can get ratio $> \pi$ only in negatively curved space – curves towards in one direction and away in another (saddle)



- If we want to do acceleration then we have to do curved spaces. ie curved spacetime!!



Triangle: $a + b + c < 180°$

Circle: Circumference $(C) > 2\pi r$

- So do we REALLY want to do acceleration ?

Gravity = Acceleration (EP)

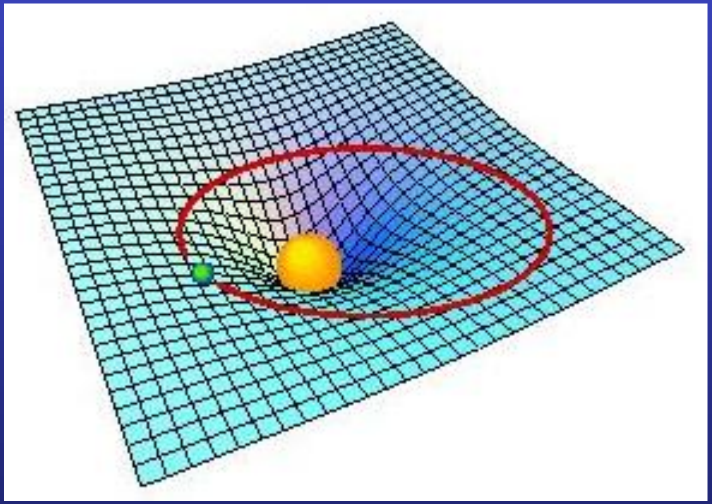
Acceleration = Curvature (SR)

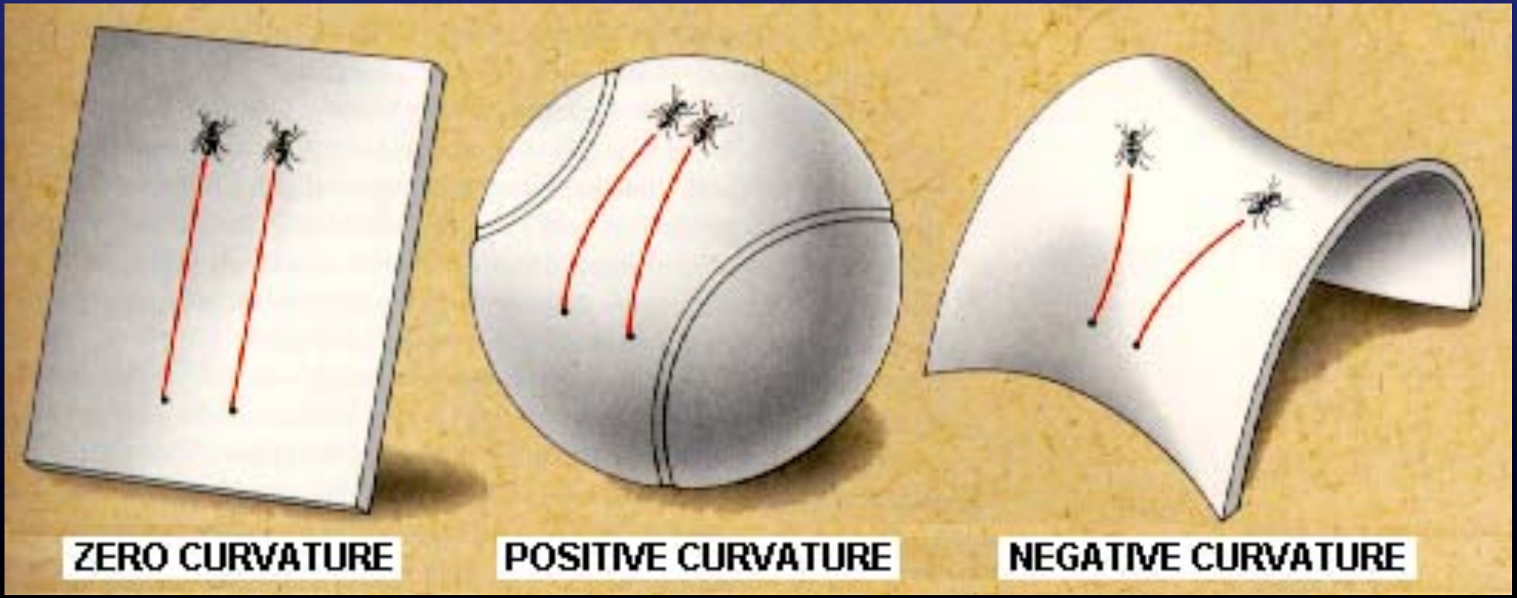
hence

Gravity = Curvature (GR)

Gravity: warped spacetime

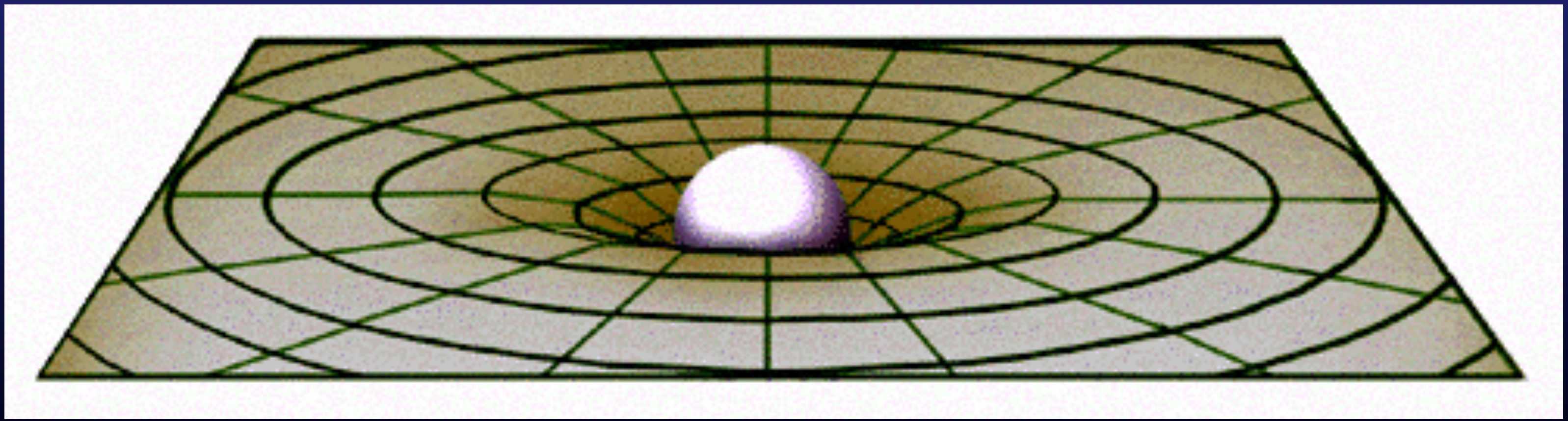
- Gravity IS curvature
- Natural paths (no forces acting ie inertial frames) are ‘straight lines’ on curved space - geodesics





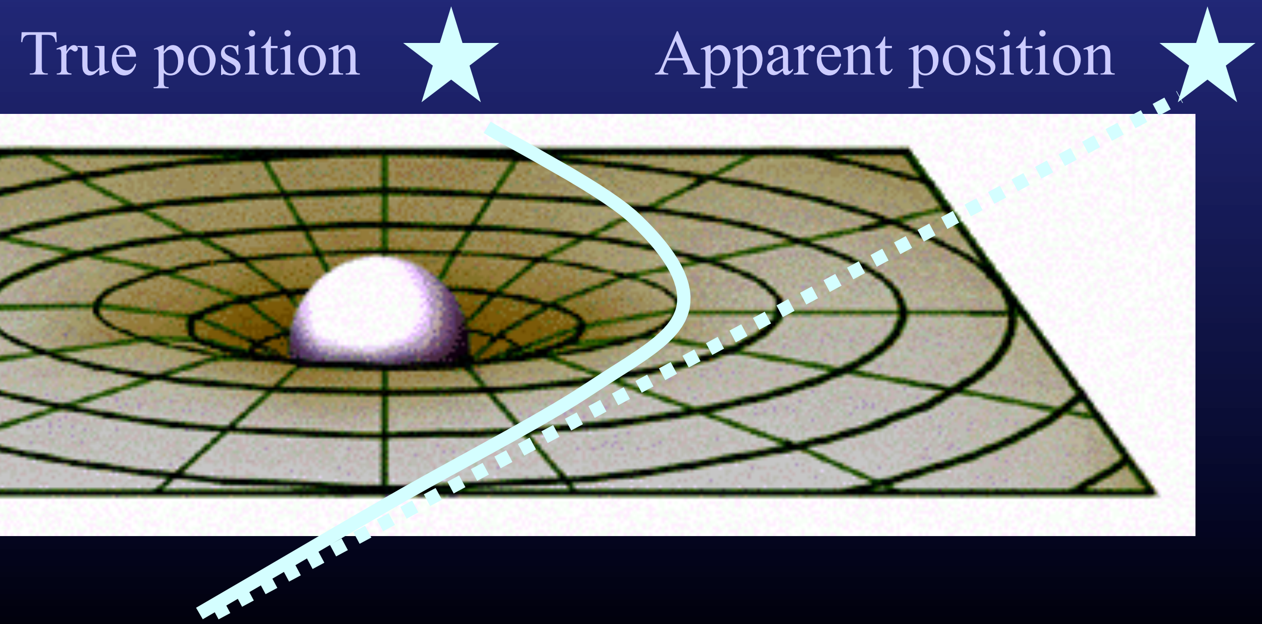
Toolkit for GR

- How to describe curvature ?
- How does mass(energy) curve space(time) ?



Toolkit for GR

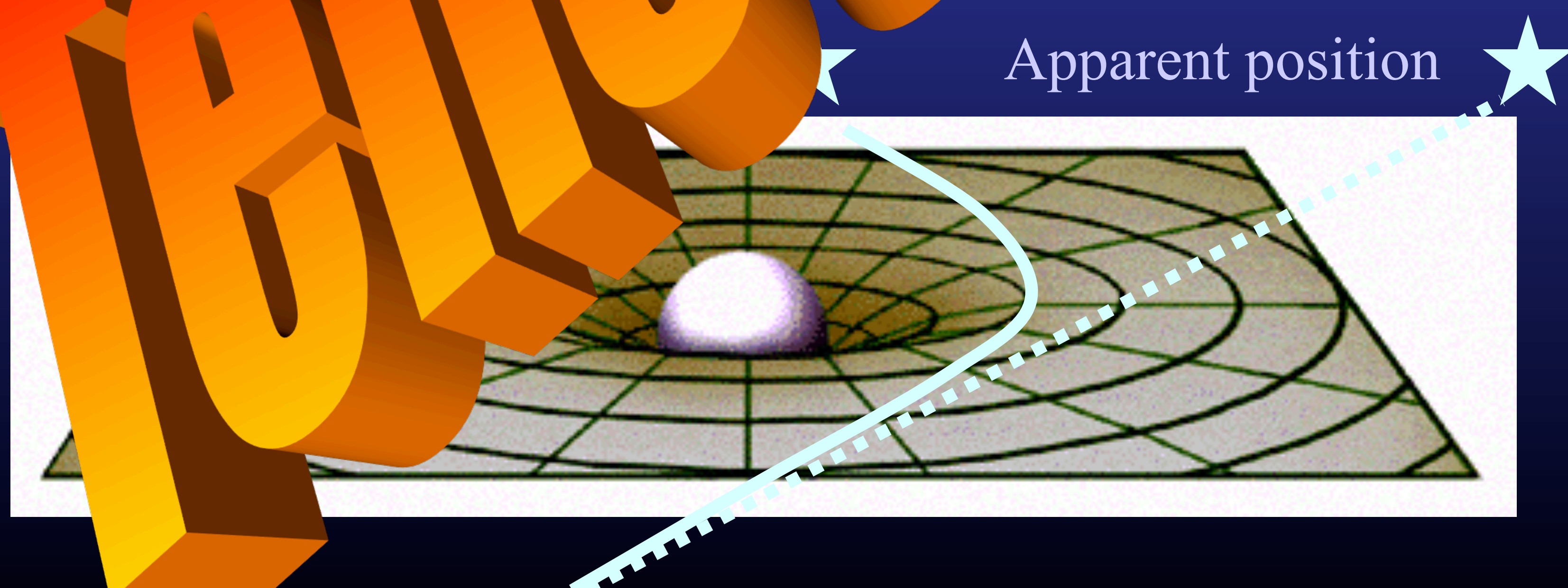
- How to describe curvature ?
- How does mass(energy) curve space(time) ?
- How to describe these ‘straight line’ natural paths?



Toolkit for GR

- How to describe curvature ?
- How does mass(energy) c[illegible]a[illegible]im[illegible]
- How to describe th[illegible]tural paths?

Tensors



Curved space tells matter how to move, matter tells space how to curve

- Understand how to describe curvature
- Find the geodesic paths on this curved spacetime. These are inertial frames so we can do physics here – SR
- Find out how energy density curves spacetime
- Requires TENSORS (don't get tense!) as this is the maths machinery developed to handle curved spaces.
- 'as simply as possible but no simpler'