

Black holes and accretion

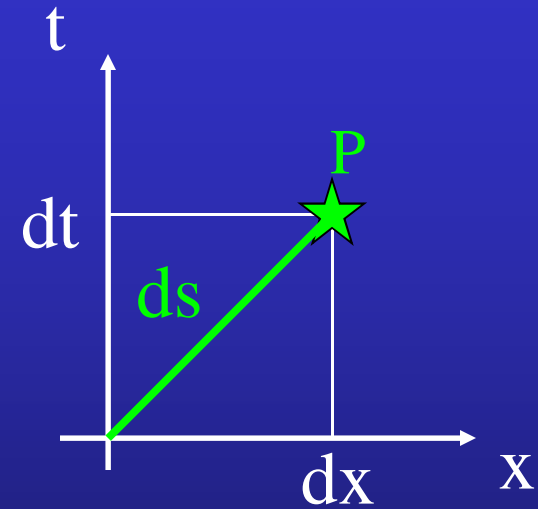
Chris Done

University of Durham

Special relativity

- Towering achievement
- Throw away ideas about fixed space and fixed time!!!
- NOT that everything is relative!
- Fixed spacetime interval
- 1D + time,

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dx^2$$



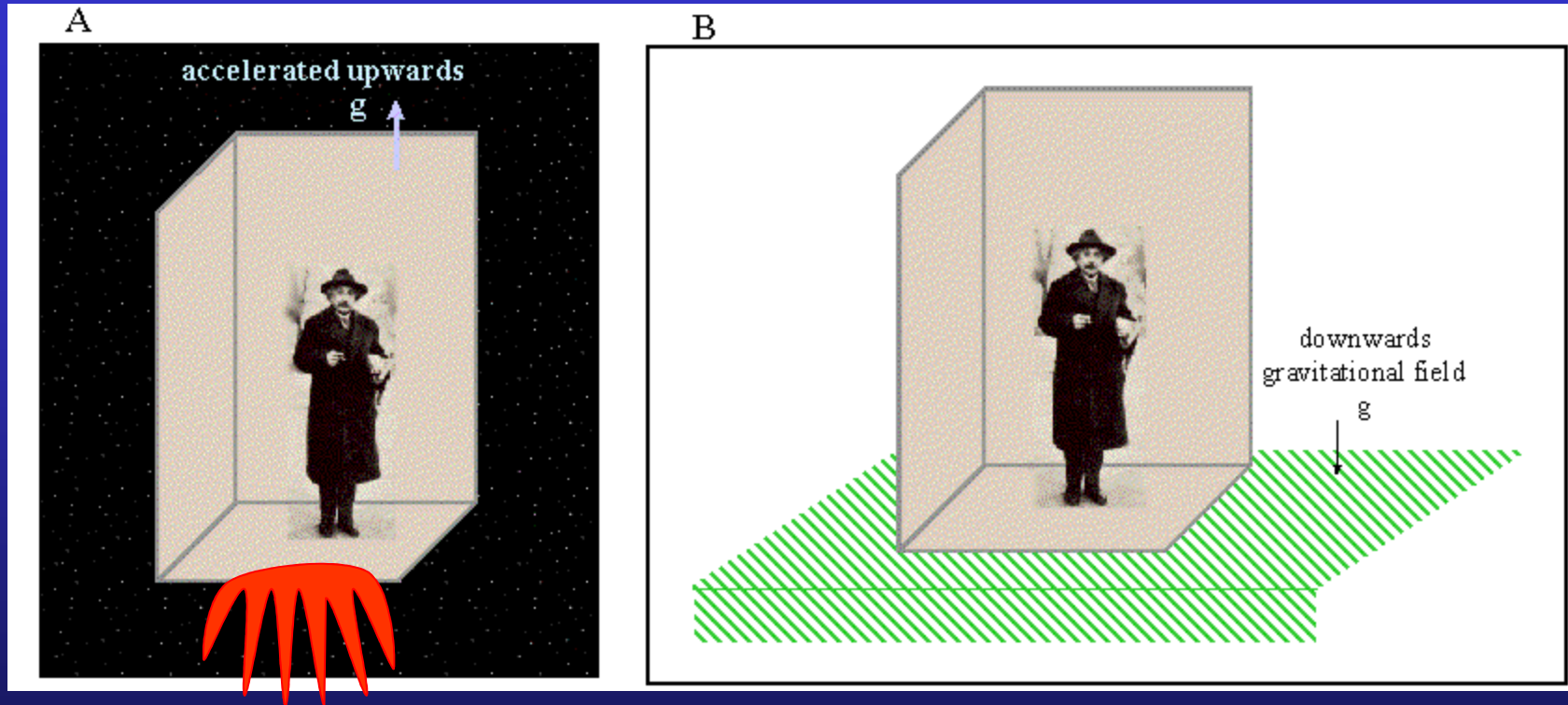
Special relativity

- $ds^2 = c^2 d\tau^2 = c^2 dt^2 - dx^2$
- Travelling at fixed speed c through spacetime....



- Generally all through time
- If put some of the motion through space then takes longer to move through time (time dilation)
- BUT only does inertial frames – not acceleration

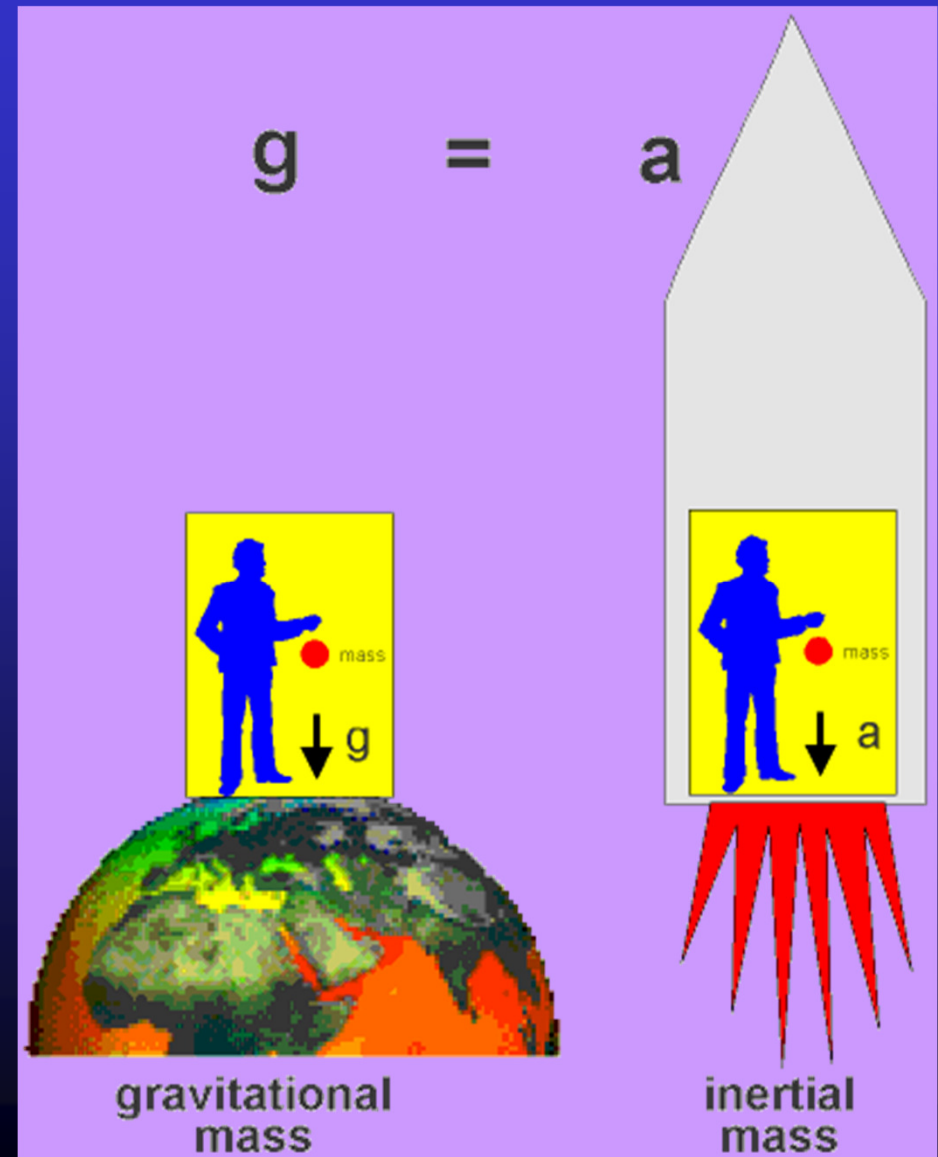
Gravity = acceleration



- difference between gravity and acceleration ?
- Look the same, behave the same...
- Maybe they ARE the same - 'happiest thought'
- Principle of equivalence: acceleration=gravity

Gravity = acceleration

- Also solves deep problem
- Inertial mass – response to accelerating force $F_i = m_i a$
- Response to gravitational force governed by ‘gravitational charge’
 $F_g = m_g GM/r^2$
- $F_g / F_i \propto m_g / m_i = \text{const}$
- No other force constant behaves like this eg EM
 $F_{em} = q Q / 4\pi\epsilon_0 r^2$
- $F_{em} / F_i \propto q / m_i$ different $e^- p^+$
- But obviously F_g / F_i same if gravity = acceleration



Acceleration: special relativity

- Circular motion easiest to think about
- Measure roundabout circumference (CL) and radius (rL) by crawling around with ruler of length L
- Get ratio $C/r=2\pi$
- Now rotate



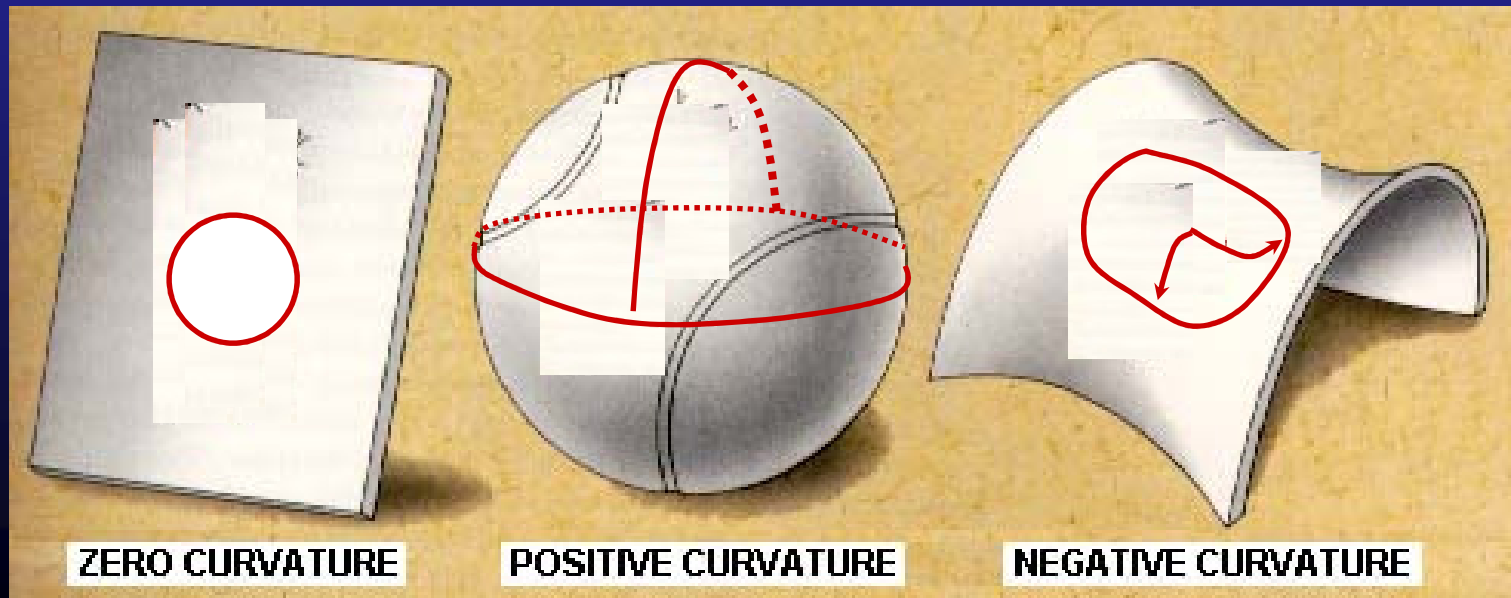
Acceleration: special relativity

- Length contracts along direction of motion so need more ruler lengths to go round $c' > c$!! But radius unaffected
- Ratio $c'/r > 2\pi$
- Can't happen!! ...in flat space

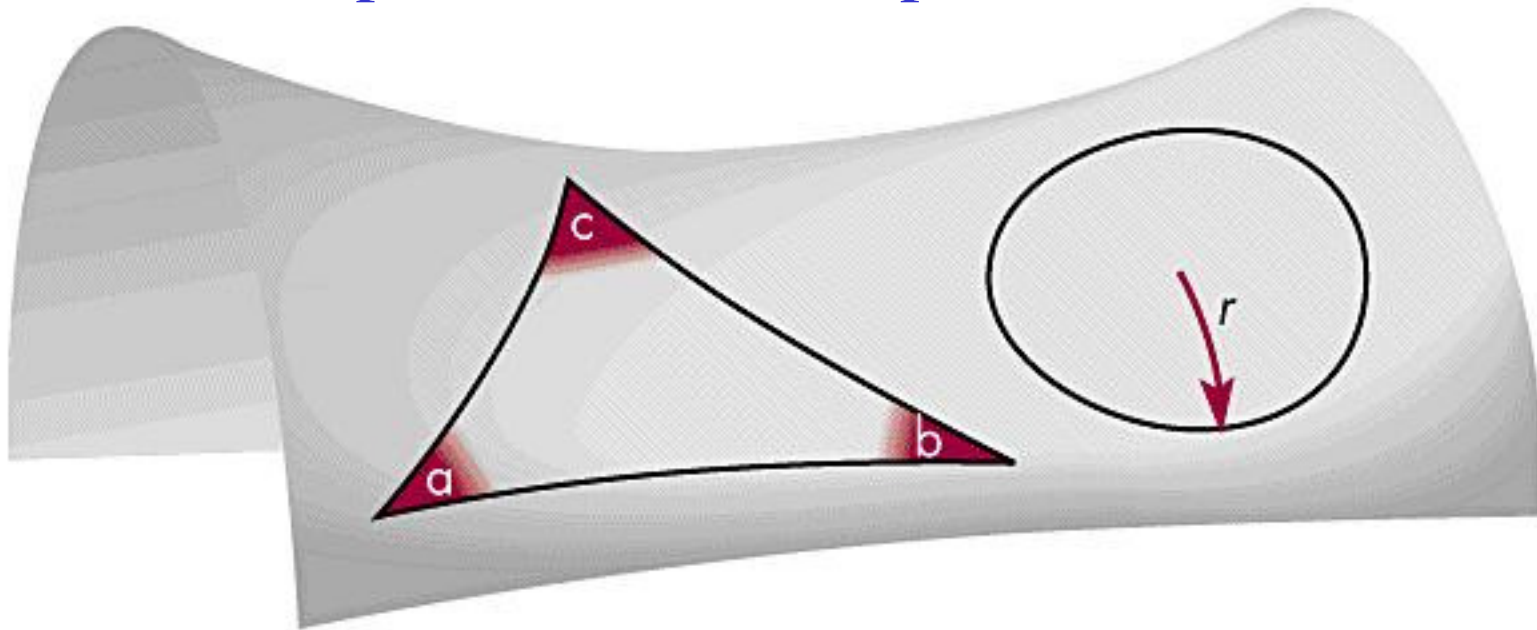


Curved spaces

- Can happen in curved spaces!!
- eg sphere. Circle round equator. Circumference is $2\pi r$, diameter is πr so ratio is $2 < \pi$!!!
- Can get ratio $> \pi$ only in negatively curved space – curves towards in one direction and away in another (saddle)



- If we want to do acceleration then we have to do curved spaces. ie curved spacetime!!



Triangle: $a + b + c < 180^\circ$

Circle: Circumference (C) $> 2\pi r$

- So do we REALLY want to do acceleration ?

Acceleration = Curvature (SR)

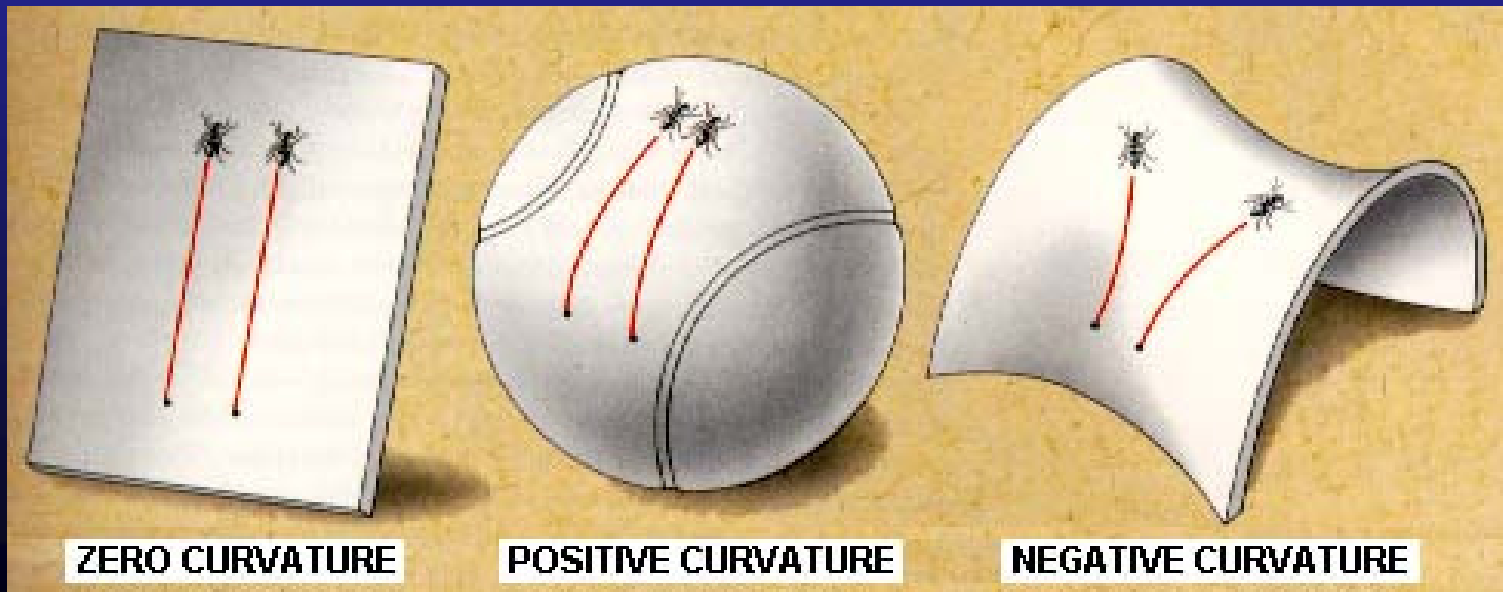
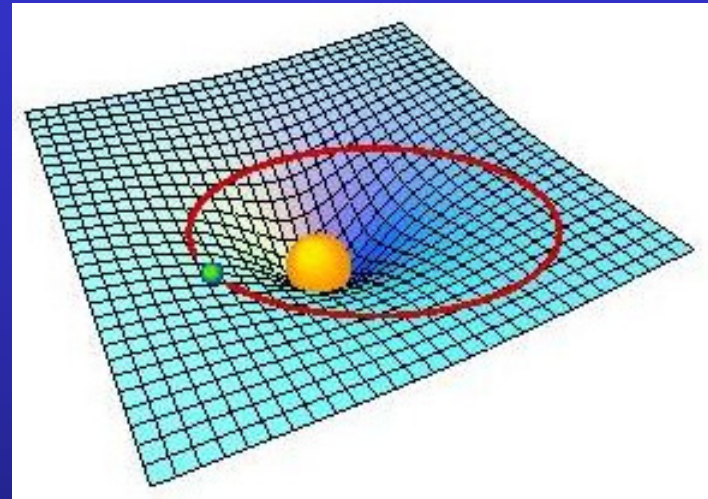
Gravity = Acceleration (EP)

hence

Gravity = Curvature

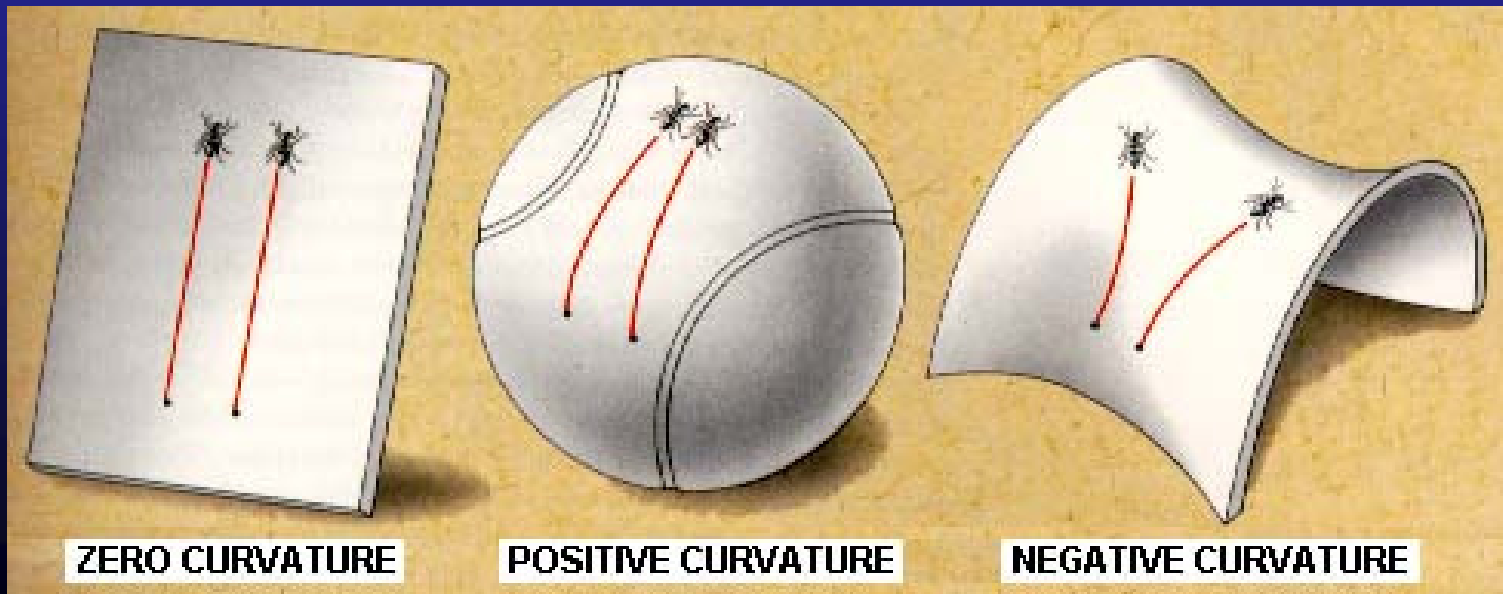
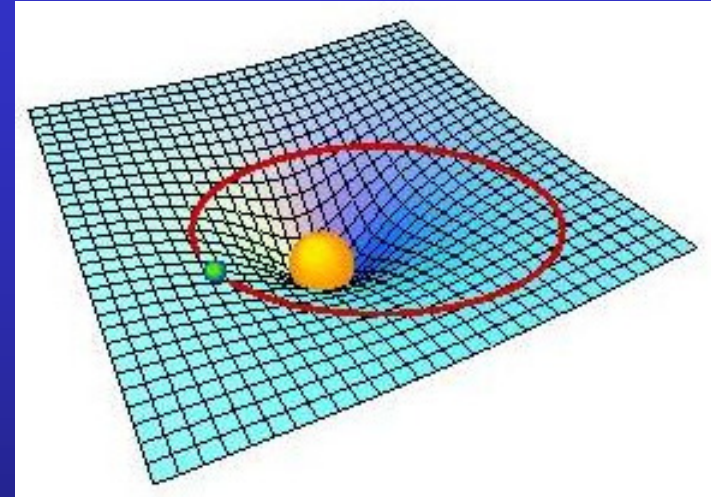
Gravity: warped spacetime

- Gravity IS curvature
- Natural paths (no forces acting ie inertial frames) are 'straight lines' on curved space - geodesics



Toolkit for GR

- Matter tells space how to curve
- Curvature tells matter how to move



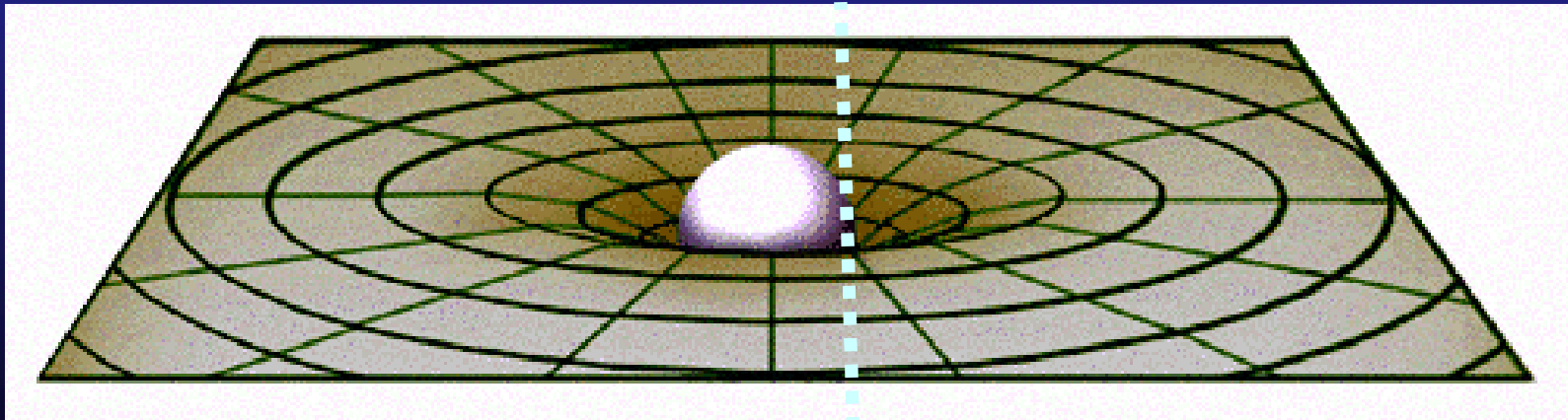
Mass and energy are equivalent

- Spacetime curved by all contributions to energy density
- Makes sense of Special Relativity!! $m=\gamma m_0$
- Increase velocity so increase KE so increase response to gravity. KE dominated by rest mass for $v \ll c$ so constant response to gravity (mass).
- But at $v \sim c$ then KE dominates. Increasing energy increases response to gravity ie increases inertial mass, so harder to increase speed!
- Stops you going faster than c

Implication: Gravity affects light

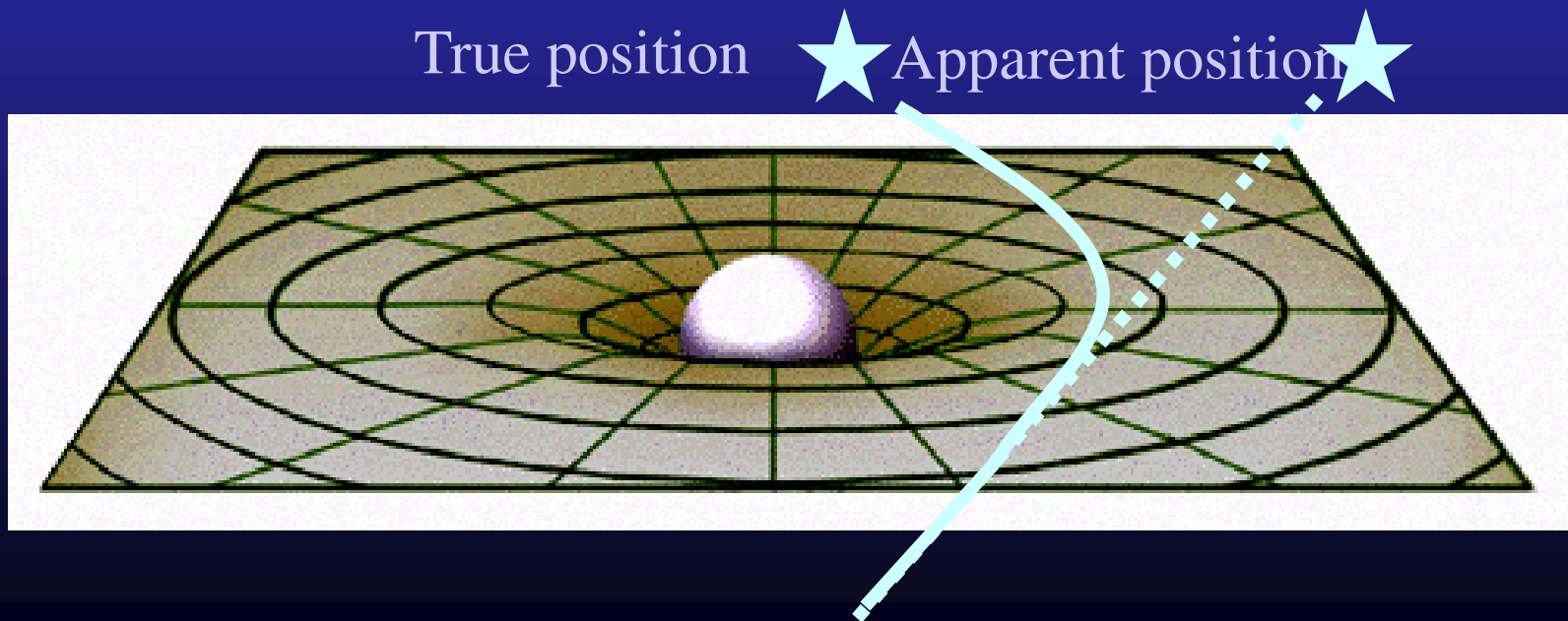
- Newton 1: $F=GMm/r^2$ so gravity doesn't affect massless particles – light travels in straight lines
- Newton 2: $F/m=a=GM/r^2$ so acceleration not dependent on mass (hammer and feather)
- GR – twice deflection of Newton 2

True position ★ Apparent position



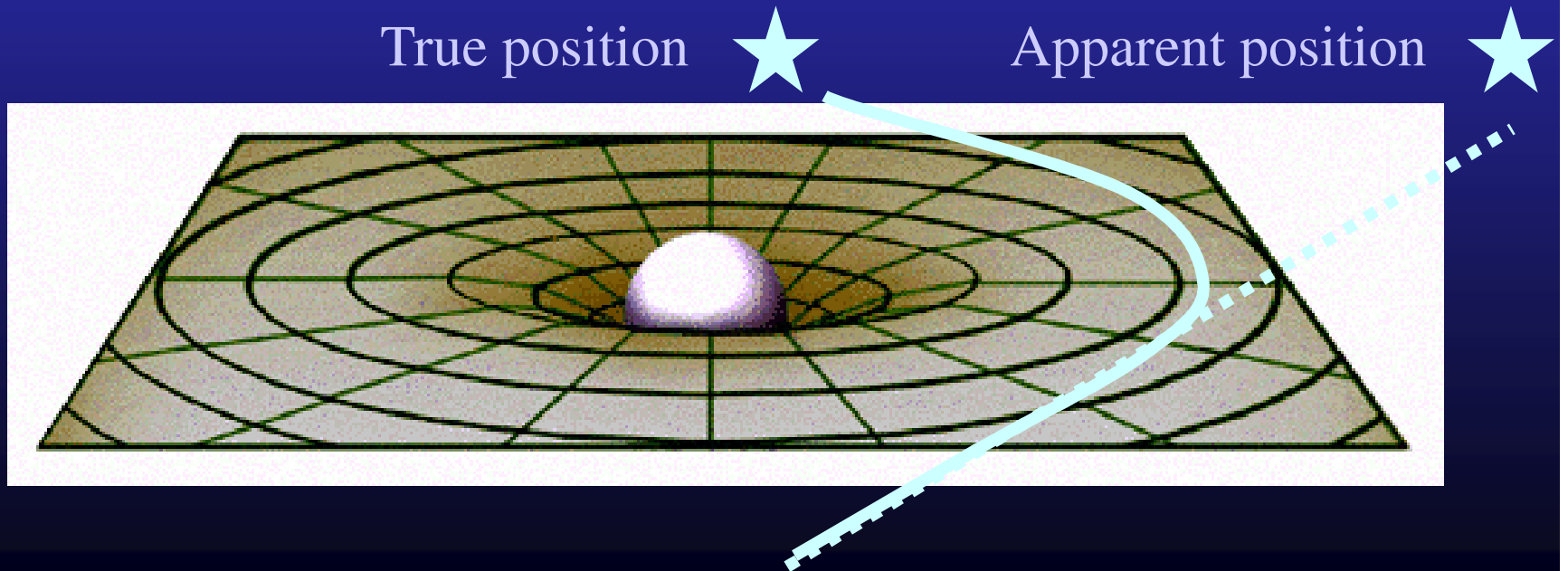
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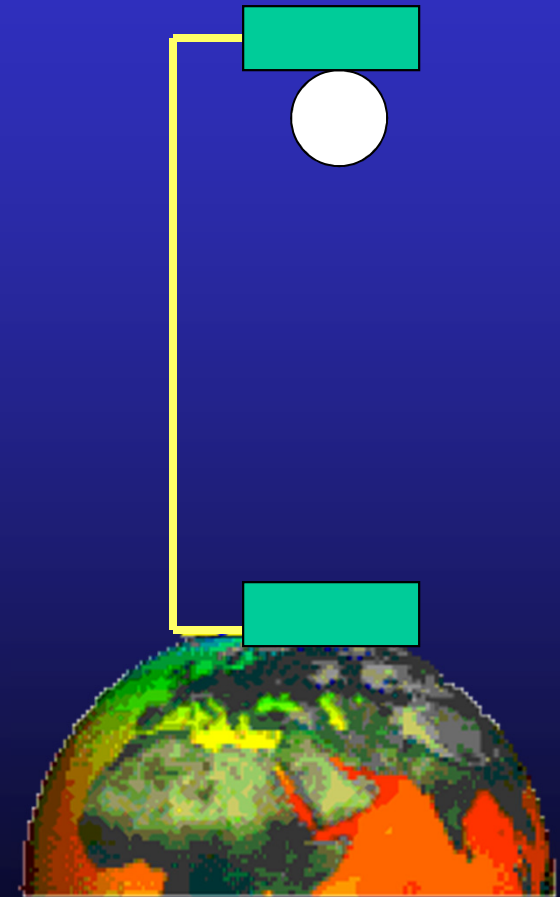
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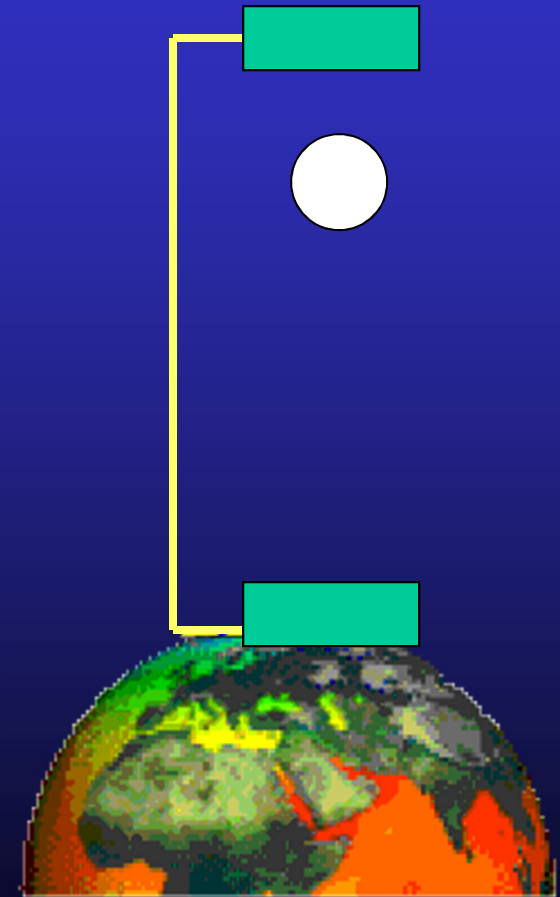
Pounds-Rebka-Snider experiment

- Drop particle from rest, mass m_1 so start from rest mass only $E=m_0c^2$



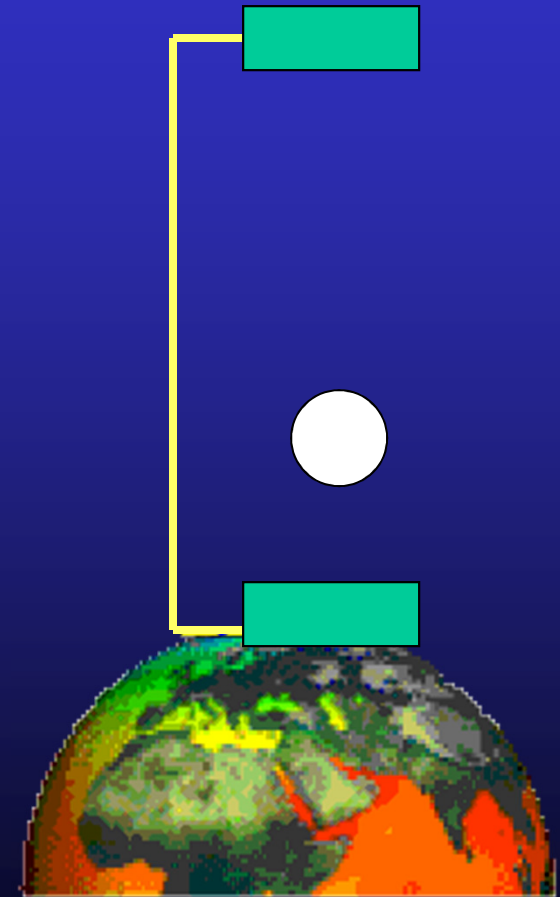
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- Gain PE: $E=m_0c^2 + m_0gh$ at base



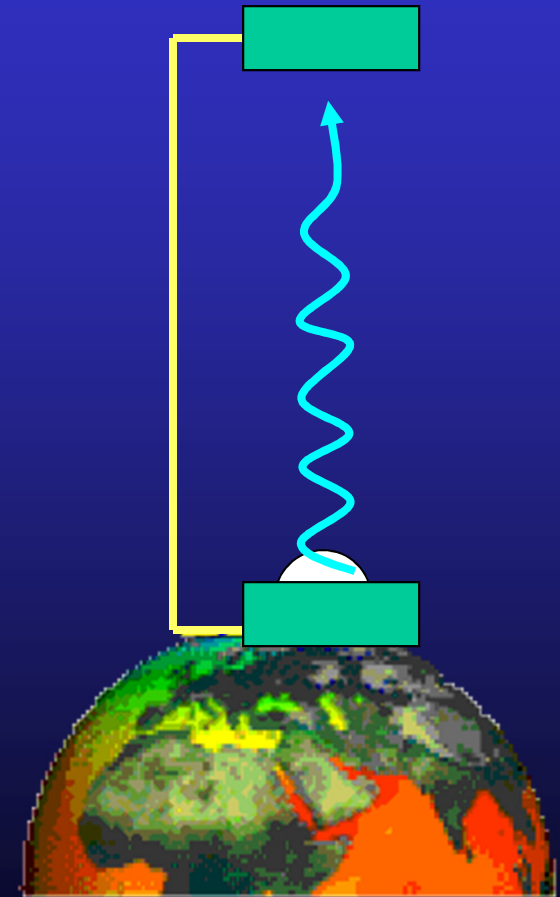
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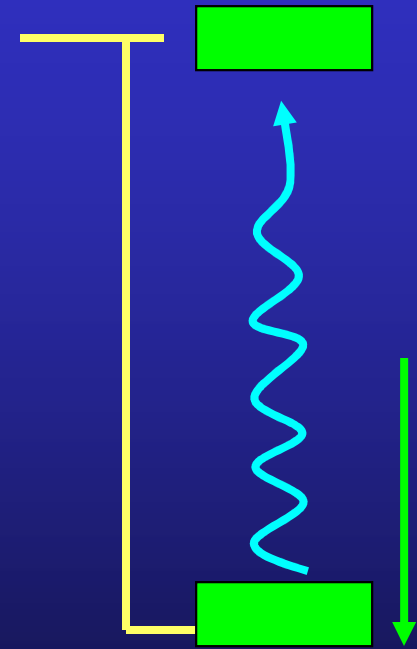
Pounds-Rebka-Snider experiment

- Convert energy to photon at base
- Send photon back up
- Convert to particle at top
- photons (massless): Newtonian
 $E = m_1 c^2 = h\nu_1 = m_0 c^2 + m_0 g h > m_0 c^2$
- Infinite energy machine !!!
- So light **MUST BE** affected by gravity. Einstein predicts this as light also travels across curved spacetime
- Time dilation = gravitational redshift
 $h\nu_t = m_0 c^2$ but $h\nu_b = m_0 c^2 + m_0 g h$
 $\nu_t / \nu_b = mc^2 / (mc^2 + mgh) \approx 1 - GMh/r^2$



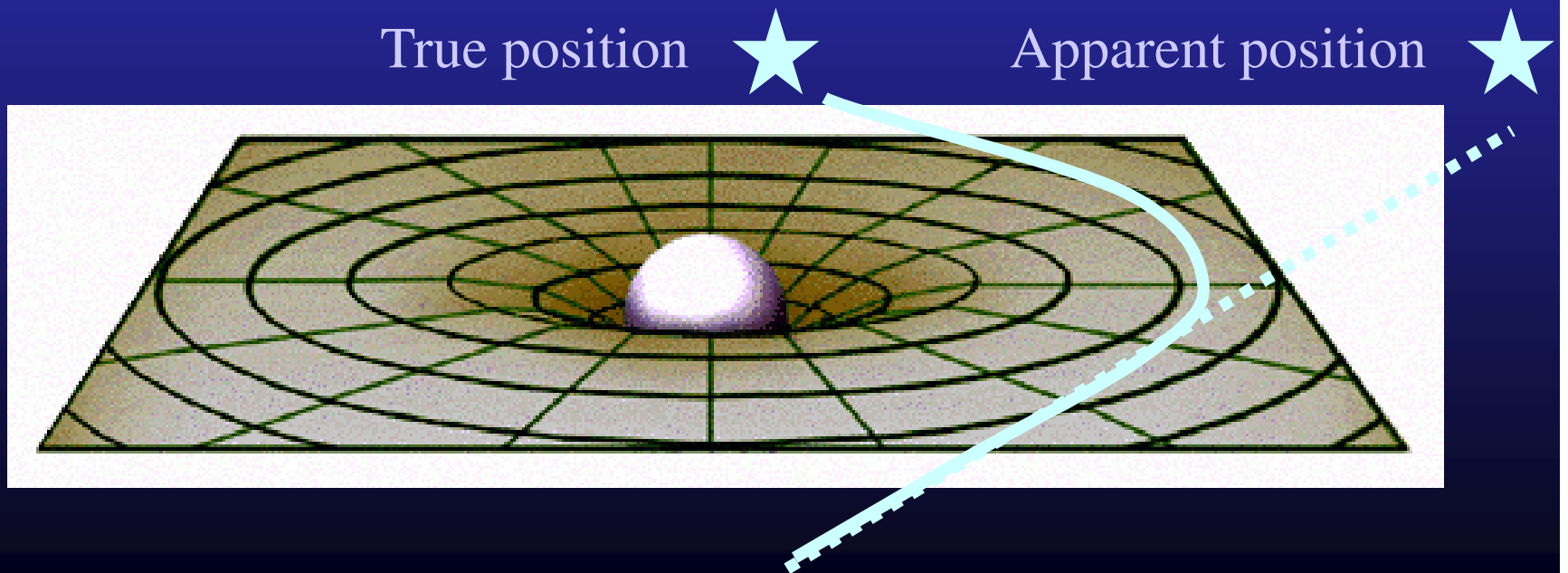
Pounds-Rebka-Snider experiment

- See the equivalence principle
- Time = h/c , acceleration $a=g$
- Speed $u=ah/c = gh/c$
- Doppler redshift $\nu/\nu_0 \approx (1-u/c)$
 $= 1-gh/c^2$ same as grav redshift
- No gravity so seen at 'top' at ν
- Same as in grav redshift!
- Free falling frames REALLY are inertial frames



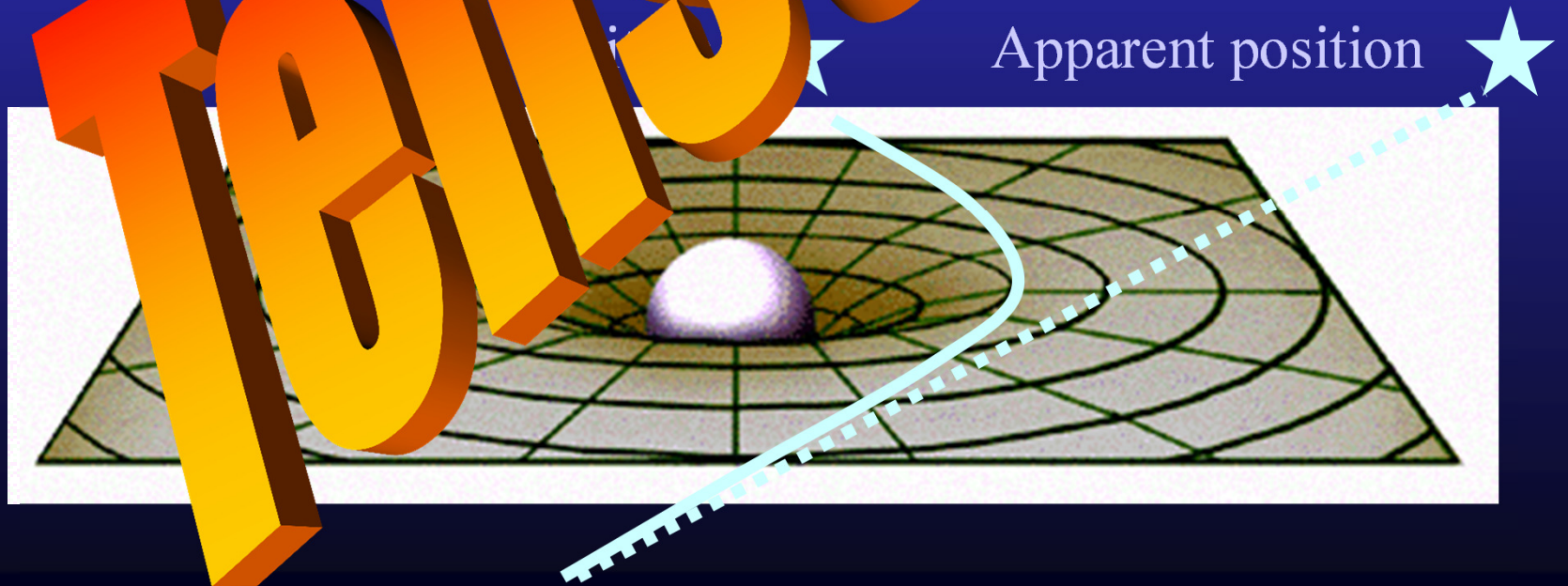
Toolkit for GR

- How to describe curvature ?
- How does mass(energy) curve space(time) ?
- How to describe these 'straight line' natural paths?



Toolkit for GR

- How to describe curvature ?
- How does mass(energy) curve spacetime ?
- How to describe these for an object's natural paths?



Curved space tells matter how to move, matter tells space how to curve

- Find out how energy density curves spacetime
- Find the geodesic paths on this curved spacetime. These are inertial frames so we can do physics here - SR
- Requires TENSORS (don't get tense!) as this is the maths machinery developed to handle curved spaces. Also easy to transform between different frames. Write physics in inertial frame: path is straight line at constant speed. Transform out and path is curved 'as if' a force is operating
- 'as simply as possible but no simpler'