lets just tidy up a few things from last week. use heisenburg uncertainty to show we can't determine which slit the photon went through... the angle to the 1st minimim is $\theta = \lambda/(2d)$. so $\Delta p_y = p_x\theta = h/\lambda \times \lambda/(2d) = h/(2d)$

we need to know $\Delta y \ll d$ in order to know which slit the photon went through - but to build up the pattern we need $\Delta p_y = h/(2d)$. so this gives $\Delta y \Delta p_y \ll dh/(2d) \ll h/2$ which doesn't work with Heisenburg!

3 Particles as waves

3.1 double slit experiement

if photons are light behaving badly (as a particle rather than a wave) then what about particles? particles behaving badly - as a wave???

yes indeed, this is exactly what we see. electrons - little bits of matter - give a diffraction pattern in the double slit experiment. this is direct confirmation that they wave wave-like properties.

eg electrons with KE of 54eV produce first maximum at an angle of 50° when scattered through Ni crystal with spacing $d = 0.215$nm which was measured from X-ray diffraction

$m\lambda = d\sin\theta$ so $\lambda = 0.165$nm.

so lets try $\lambda = h/p$ like light. but now we have $p = mv$....

$p^2/2m = KE$ so $54 \times 1.6 \times 10^{-19} \times 2 \times 9.1 \times 10^{-31} = p^2$ and $p = 3.9 \times 10^{-24}$ kg m/s $\lambda = h/p = 1.66 \times 10^{-10}$ m i.e. 0.166nm as above.

so on small enough scales, electrons act as waves not particles. which means they are like light - neither a wave nor a particle but having aspects of both.

Example: Calculate the de broglie wavelength for electrons with 1eV, 1keV, 1MeV, 1GeV.

we can use $p = mv$ for non-relativisitc, but when we get relativistic we have to use $p = \gamma mv$.

typically we say we need relativity when KE is a substantial fraction of teh rest mass energy. electron rest mass is 0.51 MeV so we are OK for the first 2, not for the second 2.

i) $\lambda = h/p$ - but given energy so use $E = p^2/2m$ or $p = \sqrt{2mE}$ and 1eV=1.6×10^{-19} J so $p = 5.4 \times 10^{-25}$ kg/m/s and $\lambda = 1.22 \times 10^{-9}$ ie 1.22nm

ii) KE=1 keV - so should be $\sqrt{10^3}\times$ smaller ie 3.8×10^{-11}m=0.038nm

iii) KE=1MeV = need $E^2 = (pc)^2 + (mc^2)^2 = (K + mc^2)^2$ so $p^2c^2 = (K + mc^2)^2 - (mc^2)^2 = K^2 + 2Kmc^2$

$K = 10^6 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-13}$ J and $mc^2 = 8.2 \times 10^{-14}$ J so $p^2c^2 = 5.18 \times 10^{-26}$ so $p = 7.58 \times 10^{-22}$ and $\lambda = h/p = 8.7 \times 10^{-13}$=0.87pm (pico=$10^{-12}$)

iv) then for $K = 1$ GeV we are very relativisitc so K much bigger than rest mass. $p^2c^2 \approx E^2 = K^2$ so $pc \approx K$

$K = 10^9 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-10}$ J so $p = 5.3 \times 10^{-19}$ kg/m/s and $\lambda = h/p = 1.23 \times 10^{-15}$ m which is 1.23fm where femto=10^{-15}

3.2 Uncertainty principle

so all the same problems as with photons - we can't determine a trajectory, only a probability.

we have the same heisenburg uncertainty principle - we can't say which slit it went through $\Delta x \Delta p \geq \hbar/2$

how long does it take a moving wavepacket to pass a particular point $\Delta t = \Delta x/v = m\Delta x/p$ but $E = p^2/(2m)$ so $\Delta E = 2p\Delta p/(2m) = \Delta p/m$

$\Delta t \Delta E = \frac{m\Delta x}{p}\frac{p\Delta p}{m} = \Delta x \Delta p \geq \hbar/2$

so the fact that there is a position-momentum uncertainty menas that there is also an energy-time uncertainty!

e.g two laser beams pointing at each other, one has wave travelling left to right, the other travelling right to left. it sets up a standing wave pattern by interference - so amplitude can be zero when the waves exactly cancel for a short time before they exactly add! so see zero energy or 2x energy on time $\Delta t \sim 1/f = h/E$.

Example: prediction of pions!

We know the strong nuclear force has a range of $R = 1.4 \times 10^{-15}$ m if this is carried by particles then $\Delta t = R/c = 4.66 \times 10^{-24}$ s so $\Delta E = \hbar/(2\Delta t) = 1.13 \times 10^{-11}$ J

$E = mc^2$ so $m = 2.25 \times 10^{-28}$ kg

which is actually pretty accurate!!