So lets work this out - for a hydrogen-like ion we have forces on circular orbits

$$\frac{mv_n^2}{r_n} = \frac{Ze^2}{(4\pi\epsilon_0 r_n^2)}$$

$$mv_n^2 = \frac{Ze^2}{(4\pi\epsilon_0 r_n)}$$

$$v_n m v_n r_n = \frac{Ze^2}{4\pi\epsilon_0}$$

$$v_n n\hbar = \frac{Ze^2}{4\pi\epsilon_0}$$

$$v_n = \frac{Ze^2}{4\pi\epsilon_0 n\hbar} = \frac{Ze^2}{2\epsilon_0 nh}$$

and then we can solve for $r_n = n\hbar/(mv_n)$

$$r_n = n\hbar\frac{2\epsilon_0 nh}{Ze^2 m} = n\frac{h}{2\pi}\frac{2\epsilon_0 nh}{Ze^2 m}$$
$$= \frac{\epsilon_0 n^2 h^2}{m\pi Ze^2} = a_0 n^2/Z$$

where $a_0 = \epsilon_0 h^2/(m\pi e^2) = 5 \times 10^{-11}$ m
so now we know KE and PE as

$$K = \frac{1}{2}mv_n^2 = \frac{1}{2}m\frac{(Ze^2)^2}{(2\epsilon_0 nh)^2} = \frac{mZ^2e^4}{8\epsilon_0 n^2 h^2}$$

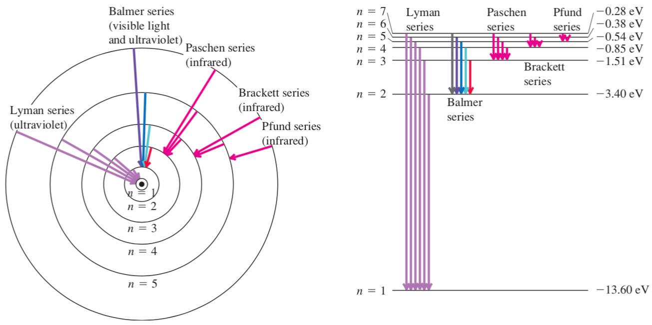
$$U_n = -\frac{Ze^2}{4\pi\epsilon_0 r_n} = -\frac{Ze^2 \times Ze^2 m\pi}{4\pi\epsilon_0\epsilon_0 n^2 h^2}$$

$$= -\frac{Z^2 e^4 m}{4\epsilon_0^2 n^2 h^2}$$

total energy

$$E_n = K_n + U_n = -\frac{Z^2 e^4 m}{8\epsilon_0^2 n^2 h^2}$$

energy difference between levels.



(a) Permitted orbits of an electron in the Bohr model of a hydrogen atom (not to scale). Arrows indicate the transitions responsible for some of the lines of various series.

(b) Energy-level diagram for hydrogen, showing some transitions corresponding to the various series

$$hf = E_i - E_f = -\frac{Z^2 e^4 m}{8\epsilon_0^2 h^2}\left[\frac{1}{n_i^2} - \frac{1}{n_f^2}\right]$$

Example: for hydrogen, $Z = 1$ and the constant is 2.18×10^{-18} but $hf = hc/\lambda$ so $1/\lambda = 1.1 \times 10^7 \left[\frac{1}{n_i^2} - \frac{1}{n_f^2}\right]$

so for transitions from $n_i = 1$ we have the Lyman series n=1-2 is $\lambda = 1.21 \times 10^{-7}$ m $= 121$ nm, and $n = 1 - 3$ is 102 nm and $n = 1 - 4$ is 97.0 mn

transitions from $n_i = 2$ is the Balmer series. so we have

$$\frac{1}{\lambda} = 1.1 \times 10^7 \left[\frac{1}{4} - \frac{1}{n_f^2}\right]$$

so $n = 2 - 3$ is 654 nm. $n = 2 - 4$ is 484 nm... and we can re-arrange

$$\frac{1}{\lambda} = 1.1 \times 10^7 \left[\frac{1}{4} - \frac{1}{n_f^2}\right]$$

$$\lambda = 363 \frac{n^2}{n^4 - 4}\ nm$$

which is pretty much exactly tha Balmer series formula that had been discovered by experiment!!

Example: size of the H atom. ground state $n = 1$ and $Z = 1$ so $r_1 = 5 \times 10^{-11}\ m$

this is called the Bohr radius of the hydrogen atom

3.7 limitations of Bohr model

Its amazing that it gets the transition energies right!! but the probability of making a transition is not given - this will set the intensity of each line

only sucessful for H-like ions. can't do He and other more complex elements. what happens when there is more than one electron?

no equation of motion - this is not a time dependent picture (standing waves!)

fundamentally its some horrid mixture of classical and quantum concepts (particle and waves) AND it doesn't conform to the uncertainty principle! The electron wave moves in a plane around the nucleus. Let's call this the xy-plane, so the z-axis is perpendicular to the plane. Hence the Bohr model says that an electron is always found at $z = 0$ its z-momentum p_z is always zero (the electron does not move out of the xy-plane). But this implies that there are no uncertainties in either z or pz, which directly contradicts $\Delta z \Delta p_z \geq \hbar/2$. so it fundamentally can't be correct. but it gives such a good match to reality that it must encompass sompething fundamental about reality!

3.8 blackbody radiation

But at least we have some undertsanding of the spectra of hydrogen, why its made up of distinct lines rather than a continuuous blackbody spectrum.

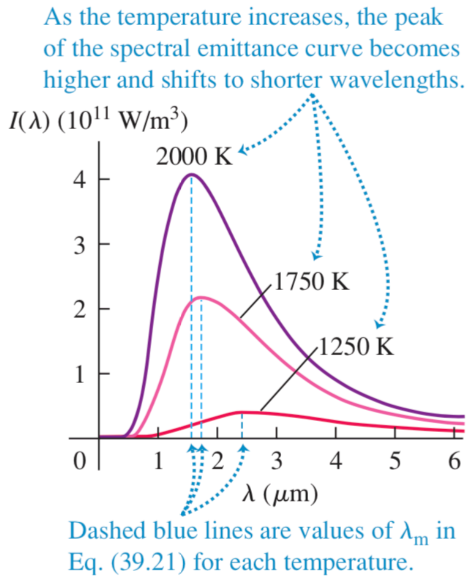
and we can understand how you go from these specific emission lines in dilute gas to blackbody from dense material by thinking about a tuning fork - a single tuning fork makes a single note, but put a whole load of tuning forks in a suitcase and their interactinos with each other mean that we get many more frequencies, not just the single specific tuning fork frequency.

The ideal surface for emitting light is one which absorbs all wavelengths of em radiation - so its black when you look at it as all radiation is absorbed! so a perfect blackbody does not actually exist as we do see heated surfaces glow red hot/white hot etc... so its an idealisation, but useful.

we know from thermodynamics that intensity $I = \sigma T^4$ where $\sigma = 5.67 \times 10^{-8}$ W/m2/K^4.

this is not uniformly distributed over all wavelengths $I = \int_0^\infty I(\lambda) d\lambda$

the peak wavelength $\lambda_m T = 2.90 \times 10^{-3}$ m K. But WHY??



3.9 Rayleigh and the UV catastrophy

em waves have magnetic and electric components. assume that they reflect off the wall of a box to set up standing waves.

the incident electric wave is $E_y(x,t) = E_{max}cos(kx - \omega t)$. This reflects to give $E_y(x,t) = -E_{max}cos(kx + \omega t)$ so we add them together to get standing waves in the y direction

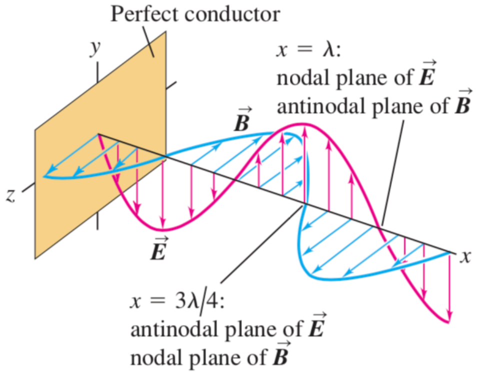
$$E_y(x,t)/E_{max} = [\cos(kx - \omega t) - \cos(kx + \omega t)]$$
$$= \cos(kx)\cos(-\omega t) - \sin(kx)\sin(-\omega t) - \cos(kx)\cos(\omega t) + \sin(kx)\sin(\omega t)$$
$$= 2\sin(kx)\sin(\omega t)$$

the E field sets up surface currents in order to cancel E on the conductor. but then the currents mean that the magnetic field is NOT zero on the conductor.

These are standing waves with $\lambda = 2L/n$ where L is the length of the box. number of standing waves in the box is $n_x n_y n_z \propto (2L/\lambda)^3$ so number per unit volume is $\propto 8/\lambda^3$. number per unit volume per unit wavelength is $\propto 8/\lambda^4$. but we want energy per unit volume per unit wavelength to match to $I(\lambda)$ units so we need an energy. Rayleigh guessed that each one had energy kT and then got (with a few constants)

$$I(\lambda) \propto \frac{8kT}{\lambda^4}$$

32.22 Representation of the electric and magnetic fields of a linearly polarized electromagnetic standing wave when $\omega t = 3\pi/4$ rad. In any plane perpendicular to the x-axis, E is maximum (an antinode) where B is zero (a node), and vice versa. As time elapses, the pattern does *not* move along the x-axis; instead, at every point the $\vec{E}$ and $\vec{B}$ vectors simply oscillate.



its actually $I(\lambda) = 2\pi ckT/\lambda^4$ when you do the long and tedious derivation.

This agrees well at long wavelengths, $\lambda \gg \lambda_{max}$. but NOT at shorter wavelengths. It just keeps on going up so that $I(\lambda) \to \infty$ as $\lambda \to 0$. This was called the ultraviolet catastropy! because you can see that there can be an infinite number of waves in the box! shorter and shorter wavelengths fit more and more easily into the box!