comparison of infinite and finite square well potentials

so its much easier to use a numerical simulation as linked on the web site for this lecture, which compares the infinite potential and a finite potential.

its clear that with the finite potential we have something like the infinite well, but with exponential tails which can extend into the classically forbidden regime ($U(x) > E$). In this region $x > L$ we found $\psi(x) \propto e^{-\kappa(x-L)} \propto e^{-\kappa l}$ where $l = x - L$ i.e. distance within the barrier. so the probability of finding the electron within a distance dx of x in this forbidden region is $\propto \psi^* \psi dx = e^{-\kappa l} e^{-\kappa l} \propto e^{-2\kappa l}$

setting $l_0 = 1/\kappa$ we see the probability $\propto e^{-l/(2l_0)}$ where l_0 is the characteristic length over which the particle extends into the forbidden region. then $l_0 \propto \hbar/\sqrt{2m(U_0 - E)}$. The bigger U_0 is compared to E i.e. the bigger the energy gap, the smaller the extent of the particle into the forbidden region.

The fact that the wavefunction extends outside of the well means that the wavefunction in the well doesn't have to be so highly curved i.e. its wavelength is bigger for any given n, so its energy is lower than the same n in the infinite well. $E_{n,finite} < E_{n,\infty}$.

For any n, we have to join the exponential tails smoothly onto the bound oscillatoray solution. so for odd n both tails are $\propto +e^{-\kappa l}$ whereas for even n they have +ve sign on one side, and -ve sign on the other. but remember probability doesn't care about the sign in front of the wavefunction (or whether its real or complex!). probability $\propto |\psi(x)|^2$

The animation is done in units of $E_{1,\infty}$

The finite well depth also means that there are only a finite number of bound states, instead of the infinite number in the infinite well. this becomes very obvious when U_0 is only a few times larger than the ground state of the infinite square well

e.g. if $U_0 = 10.13E_{1,\infty}$ (they call this $4V_0$) then $E_1 = 0.69E_{1,\infty}$ and there are 4 bound states.

e.g. if instead $U_0 = 2.52E_{1,\infty}$ (they call this V_0) then $E_1 = 0.5E_{1,\infty}$ and there are only 2 bound states.

in the limit where $U_0 \ll E_{1,\infty}$ then there is only one single bound state with $E_1 = 0.68U_0$