1.2 Postulates of special relativity YF37.2-37.3

Einstein's reaction to the failure to detect an 'aether' was radical. He embraced it wholeheartedly!

Einstein 1: the laws of physics are the same in every inertial frame of reference (this is in the spirit of Newtonian physics, as we saw from last lecture)

Einstein 2: the speed of light in a vacuum is the same in all inertial frames of reference and is independent of the motion of the source (this takes Maxwells equations at their word! so in some sense we didn't need to say it as its just a specific case of a law of physics in the 1st postulate - but its good just to be explicit because the implications are profound.)

We saw last lecture that the speed of light being constant means that the 'obvious' way in classical mechanics to transform velocties between inertial frames is not correct. Velocity is distance/time so it could be distances that go funny, or time that goes funny (or both).

1.3 ultimate speed limit

The first thing that this tells us is that the speed of light is an ultimate speed limit, that no inertial observer can travel at c. Suppose S' moves with velocity relative to S of c. Then turns its headlights on. in S, the headlight must travel at c. but so does the spacecraft, so the light is always in the same place as the spacecraft. Yet in the spacecraft in S' they have to see it move away from the spacecraft at c, so they cannot be at the same point in spacetime. as a 16 year old Einstein wondered what he would see if he were travelling at c - but the answer is you can't.

1.4 Simulteneity

The speed of light being constant has another odd implication. Events that are simultaneous for one observer need not be simultaneous for another.

So lets look at who sees what for multiple lightning strikes. Example YF37.5. An observer outside the train sees lightning strike points A and B simultaneously, burning both the ground and the front and back of the carriage. But the observer in the train is travelling forward so sees the light from the lightning strike the front of the train first (remember the light speed is the same measured in both frames!)

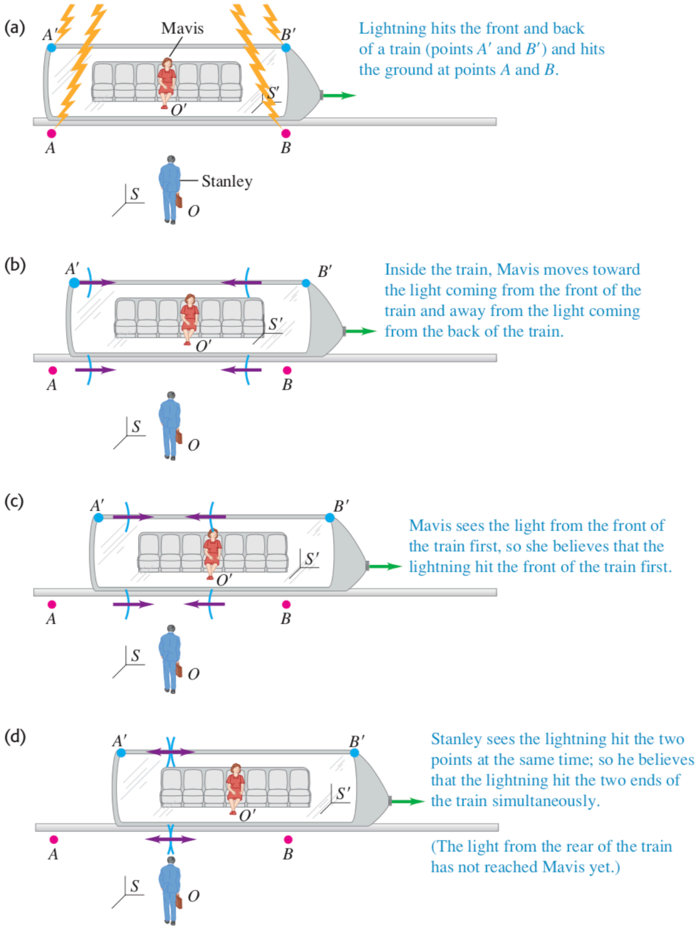
(a)
Mavis
A′
B′
S′
O′
A
B
Stanley
S
O
Lightning hits the front and back of a train (points A′ and B′) and hits the ground at points A and B.
(b)
Inside the train, Mavis moves toward the light coming from the front of the train and away from the light coming from the back of the train.
(c)
Mavis sees the light from the front of the train first, so she believes that the lightning hit the front of the train first.
(d)
Stanley sees the lightning hit the two points at the same time; so he believes that the lightning hit the two ends of the train simultaneously.
(The light from the rear of the train has not reached Mavis yet.)

Figure 1:

Always a problem here is that SIMULTENEITY is not the same as CAUSALITY. everyone measures causal events - which require that they happen at the SAME point in space AND time - to happen in the same order. The lightning strike at the front does not CAUSE the lighting strike at the back. its just viewpoint changing. If instead we were looking at lightning hitting the front of the train, which casues a flash of light that hits the back of the train causing an arc lamp to discharge and burn the wheel and tracks, then everyone will see these events in this order!

CAUSALITY is physics, and it always works. Simulteneity is not - its more like whether I see a firework explode just as it is along my line of sight to eifel tower - if I was standing somewhere different I'd see it explode to the right of the tower, or shift over and I'd see it explode to the left of the tower. only if the firework was ATTACHED to the tower would every observer position agrees that it explodes on the tower. We are very used to this for space, its not odd at all. The problem is we are really really not used to this for TIME.

1.5 relativity of time intervals - time dilation

So now lets do time intervals more formally. We have observers in frame S, and one in S'. S' moves with velocity u relative to S, along the x-x' direction and their axes O and O' align at t=t'=0.

Set off a beam of light at O' in S' - verticaly upwards and reflect it from a mirror. In S' then this takes time $\Delta t' = 2d/c$

In S, then the light path is not vertical, but moves horizontally by $u\Delta t$ as well as vertically. so then the path length up to the mirror is ℓ where $\ell^2 = d^2 + (u\Delta t/2)^2$, and the total path length is 2ℓ.

Then because light always travels at c this means the time interval is

$$\Delta t = 2\ell/c = \frac{2}{c}\sqrt{d^2 + \left(\frac{u\Delta t}{2}\right)^2}$$

square this and get

$$\Delta t^2 = \frac{4}{c^2}\left[d^2 + \frac{u^2\Delta t^2}{4}\right] = \frac{4d^2}{c^2} + \frac{u^2}{c^2}\Delta t^2$$

but $\Delta t' = 2d/c$ and $\beta = u/c$ then $\Delta t^2 = (\Delta t')^2 + \beta^2\Delta t^2$ and hence

$$\Delta t = \frac{\Delta t'}{\sqrt{1-\beta^2}} = \frac{\Delta t'}{\sqrt{1-u^2/c^2}}$$

37.6 (a) Mavis, in frame of reference S', observes a light pulse emitted from a source at O' and reflected back along the same line. (b) How Stanley (in frame of reference S) and Mavis observe the same light pulse. The positions of O' at the times of departure and return of the pulse are shown.

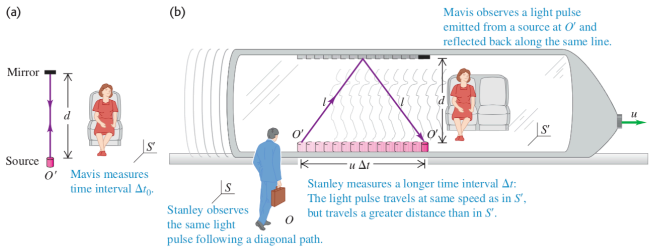

Figure 2:

since $u/c < 1$ then $\Delta t > \Delta t'$.

the time for the light pulse to go up and down is longer when we are NOT in the frame in which the light pulse is emitted. Fast clocks run slow

suppose we have $d = 1.5$m and $u = 300$m/s (more like a plane than a train!) then

$\Delta t' = 2d/c = 10^{-8}$s

$\Delta t = \gamma \Delta t'$

if $u/c = \beta \ll 1$ we can use the approximation $(1 + x)^n = 1 + nx + \ldots$ where $\gamma = (1 - \beta^2)^{-1/2}$ so $x = -\beta^2$ and $n = -1/2$ so $\gamma \approx 1 + (-\beta^2)(-1/2) = 1 + \beta^2/2 = 1 + (300/3 \times 10^8)^2/2 = 1 + 10^{-12}/2$ This is a TINY effect.

But now take $u = 0.99c$ then $\gamma = 7.1$ and $\Delta t = 7 \times 10^{-8}$ s

or $u = 0.9999c$ and $\gamma = 70.7$ and $\Delta t = \gamma \Delta t' = 7 \times 10^{-7}$ s

1.5.1 Proper time

there is only one inertial frame in which we are in the same frame as an event, but infinitely many which are moving. So time intervals measured in the same frame as the event have a more fundamental quality than those in any other frame. we use the term **proper time**, T_0, to describe the time interval between 2 events which occur at the same point.

37.8 The quantity $\gamma = 1/\sqrt{1 - u^2/c^2}$ as a function of the relative speed u of twc frames of reference.

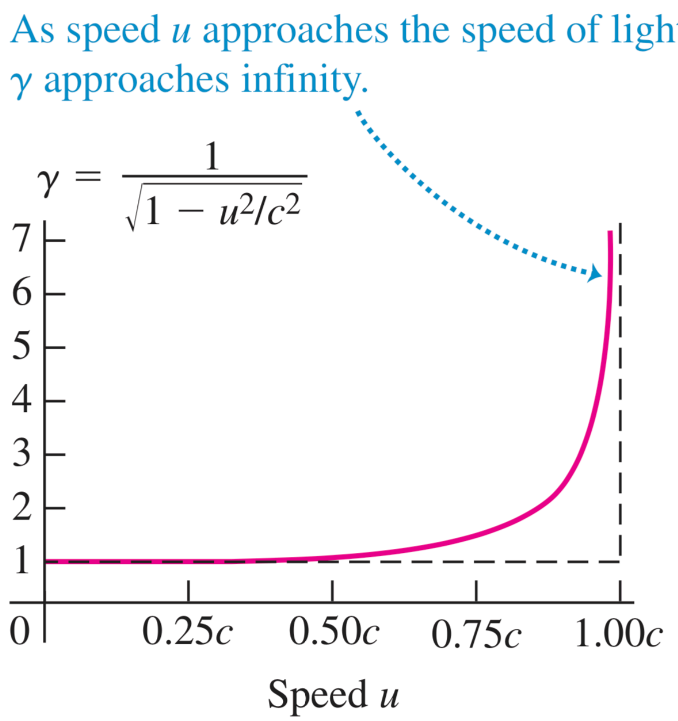

Figure 3:

(I think this is slightly confusing as in the standard setup of S and S', proper time is measured in S' - so I'd expect it to have a prime on it!)

Proper time is always the shortest, all other frames, S, see S' move with velocity u so they measure time intervals which are longer by a factor $\gamma(u)$

1.5.2 Examples

e.g. example YF37.1 high energy particles from space interact with atoms in the earths upper atmosphere to produce muons. These decay in their rest frame with lifetime $\Delta t' = 2.2 \times 10^{-6}$ s.

if the muon is moving with respect to the earth with $u = 0.99c$ then what is its mean lifetime as measured on earth?

$\gamma = 1/\sqrt{1-\beta^2} = (1-0.99^2)^{-1/2} = 7.09$ so $\Delta t = 7.09 \times \Delta t' = 1.56 \times 10^{-5}$ s.