8 Nuclear physics

basic properties of nucleons: proton mass 938.28 MeV/c^2 ($1.007276u$) whereas neutron is 939.57 MeV/c^2 ($1.008665u$) where $u = 1.66{\times}10^{-27}$ kg $= 931.5$ MeV/c^2. proton is stable (or huge lifetime) whereas free neutrons decay in 918 s (to proton and electron and neutrino). charge is $+e$ versus 0, spin is $s = 1/2$ for both (fermions).

spin means there is a magnetic moment if there is charge. for electrons, we quantised in terms of $\mu_B = e\hbar/(2m_e)$ (Bohn magneton). Now a more natural unit is $\mu_n = e\hbar/(2m_p)$ (nuclear magneton). We might then expect that proton magnetic moment about the z-axis is $\mu_z = m_s\mu_n$ - but actually its $2.8\mu_n$, and we might expect that the neutron spin magnetic moment is zero as it has no charge. but its not, its similar in size at $1.9\mu_n$, but opposite in sign. This shows its NOT actually an elementary particles - elementary particles MUST have charge as well as spin to have a magnetic moment - and that neutrons (and protons) are made up of quarks.

Z is atomic number which is the number of protones, $A = Z + N$ is mass number (sum of number of protons Z and number of neutrons N).

In detail, mass is NOT the same as mass number due to binding energy $E_B = (Zm_p + Nm_n - m)c^2$. But its approximately right $m \approx Am_p$.

Any nucleus can be written as ^A_ZSymbol, where symbol is H, He, Li etc denoting the element name e.g $^{12}_6C$

a nuclear species is a nuclide

An isotope is a nucleus with the same number of protons, but different number of neutrons e.g. $^{12}_6C$, $^{13}_6C$ and $^{14}_6C$.

An isotone is a nucleus with the same number of neutrons but different number of protons e.g. ^{115}B $^{12}_6C$, $^{13}_7N$.

An isobar has the same mass number $^{13}_6C$ and $^{13}_7N$

8.1 nuclear radii and densities

to a good approximation, the nuclear density is the same for all nucleons. $\rho = Am_p/(4/3\pi R^3)$, so radii are $R \approx R_0 A^{1/3}$ for $R_0 = 1.2\times 10^{-15}$ m = 1.2 fm

example YF43.1: ${}^{56}_{26}Fe$ is the most common iron isotope. Find its radius, approximate mass, and approximate density of the nucleus

$R \sim R_0 A^{1/3} = 1.2 \times 10^{-15} 56^{1/3} = 4.6 \times 10^{-15}$ m=4.6 fm

$m \approx 1.66 \times 10^{-27} A \sim 9.3 \times 10^{-26}$ kg

$\rho = m/V$ and $V = (4/3)\pi R^3 = 1.333 \times \pi \times (4.6 \times 10^{-15})^3 = 4.1 \times 10^{-43}$ m^3 so $\rho = 9.3 \times 10^{-26}/4.1 \times 10^{-43} = 2.3 \times 10^{17}$ kg/m^3

the density of solid iron is 7000 kg/m^3, which is more than $10^{13}\times$ lower!

8.2 nuclear mass (detailed)

binding energy $E_B = (Zm_p + Nm_n - m)c^2$ where m is the mass of the neutral atom containing the nucleus. E_B/c^2 is called the mass defect.

To get the binding energy/mass defect we have to be accurate as it involves subtracting two numbers which are almost equal, the sum $Zm_p + Nm_n$ versus m.

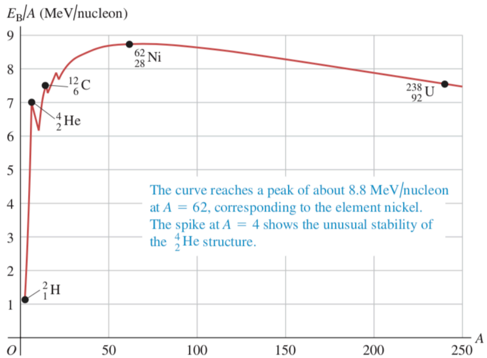
example deuterium ${}^2_1H$. 1 proton and 1 neutron. measured mass of neutral atom 2.014102u.

$E_B = (1.007825 + 1.008665 - 2.014102)uc^2 = 2.388 \times 10^{-3} uc^2 = 2.224\ MeV$

This is how much energy is required to pull apart the neutron and proton.

binding energy per nucleon is $E_B/A = 1.112$ MeV for deuterium.

Do this for all the stable nuclei and plot E_B/A for all of them. Deuterium is the lowest, ${}^{62}28Ni$ is the highest. but its NOT smooth. there is structure.



The curve reaches a peak of about 8.8 MeV/nucleon at $A = 62$, corresponding to the element nickel. The spike at $A = 4$ shows the unusual stability of the $^{4}_{2}$He structure.

Figure 2:

Clear peaks when number of neutrons or number of protons are 2,8,20,28,50,82,126. And even better when BOTH neutrons and protons have these numbers e.g at ^4_2He and $^{16}_8O$ and $^{40}_{20}Ca$ and $^{48}_{20}Ca$ All these have substantially higher binding energy than nuclei with neighbouring values of N, Z. This is showing us something about the energy level structure of the nucleus, with filled shells like for electrons in atoms giving very stable chemical structure when the shells are full (He, Ne, Ar etc.).

Difference in E_B/A for He and above not large - $6.2 - 8.8$ MeV/nucleon.

8.3 Nuclear force

There is very strong electrostatic repulsion between protons in the nucleus as the distances are very small. The force which can bind them together despite this is the strong nuclear force. It does not depend on charge (affects neutrons as well as protons), and has very short range, of order the nuclear size of 10^{-15} m (otherwise it would pull in more nucleons), but within its range it is stronger than the electrostatic repulsion (otherwise nuclei could not be stable!). T he nearly constant density and nearly constant E_B/A means that a particular nucleon only interacts with those in its immediate vicinity, not all of them - saturation in number of nucleons which take part in the interaction.

It favours pairs of neutrons and protons, and especially pairs of pairs eg ${}^4_2He$. these pairs are opposite spin states. so even numbers of protons and even numbers of neutrons are a term.

But we still can't write down its potential and solve the Schroedinger equation! the liquid drop model is a semi-empirical model for E_B which is able to fit in a fairly smooth way to E_B and tells us about the required strength of some of the component parts of the potential. But this doesn't give us the detailed peaks of E_B so its missing something about the filled shells.

8.3.1 Shell model

The shell model is analogous to the central field model in atomic physics for multi-electron atoms. assume each nucleon moves in the mean field of every other nucleon.

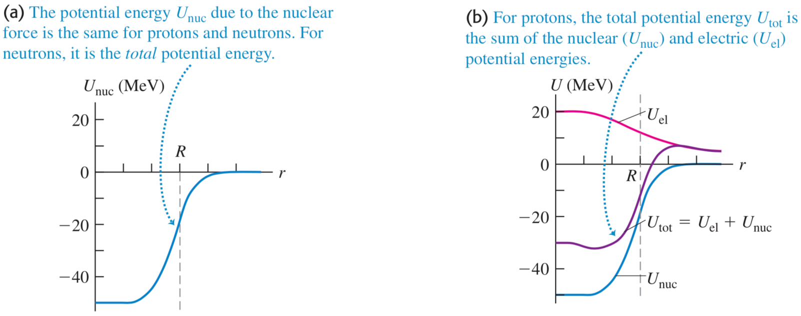
The potential is DIFFERENT for neutrons and protons as protons see the electrostatic repulsion as well as the strong force attraction.

But both neutrons and protons have $s = 1/2$ so we can only fit at most 2 protons in any one state for the proton potential, and at most 2 neutrons in any one state for the neutron potential. So its like in atoms where we have the pauli exclusino principle and we have to fill up states starting at the ground level and going up till we have fit all the particles in.

The repulsion does not keep on increasing as the charge is distriubuted throughout the nucleaus and so the charge enclosed drops as you go inwards (Gauss's law).

But both neutron and proton potentials look (kind of) like a 3D harmonic potential! so we can use this as a model to predict the shell structure. Remember this had LOTS of degeneracy. $E = (n_x + n_y + n_z + 3/2)\hbar\omega$ where $\omega = \sqrt{k'/m}$

ground state $n_x = n_y = n_z = 0$ $E(0, 0, 0) = 3/2\hbar\omega$ only one way to get this energy - degeneracy 1 (or non-degenerate) but 2 spin states so 2 neutrons. or 2 protons, so most stable is two of BOTH i.e. ${}^4_2$ He.



(a) The potential energy U_{nuc} due to the nuclear force is the same for protons and neutrons. For neutrons, it is the *total* potential energy.

(b) For protons, the total potential energy U_{tot} is the sum of the nuclear (U_{nuc}) and electric (U_{el}) potential energies.

Figure 3:

1st excited state $n_x = 1$, $n_y = n_z = 0$ (1,0,0). SAME ENERGY as $n_x = 0$, $n_y = 1$, $n_z = 0$ (0,1,0). SAME ENERGY as $n_x = 0$, $n_y = 0$, $n_z = 1$ (0,0,1). $E = (1 + 3/2)\hbar\omega = 5/2\hbar\omega$ Three ways to get this energy - degeneracy 3. but 2 spin states so 6. and we get another filled shell for 6 neutrons (and 6 protons). so another filled shell for 8 protons and/or 8 neutrons i.e. O

2nd excited state $n_x = 2$, $n_y = n_z = 0$ (2,0,0). SAME ENERGY as $n_x = 0$, $n_y = 2$, $n_z = 0$ (0,2,0). SAME ENERGY as $n_x = 0$, $n_y = 0$, $n_z = 2$ (0,0,2). SAME ENERGY as $n_x = 1$, $n_y = 1$, $n_z = 0$ (1,1,0). SAME ENERGY as $n_x = 1$, $n_y = 0$, $n_z = 1$ (1,0,1). SAME ENERGY as $n_x = 0$, $n_y = 1$, $n_z = 1$ (0,1,1). $E = (2 + 3/2) = 7/2\hbar\omega$. six ways to get this energy - degeneracy 6. but 2 spin states so another filled shell for another 12 nucleons. so total of 20 protons and/or neutrons. These match the peaks on the binding energy curve!