Things can get tricky. but generally only because they are described in a confusing way.

example YF 37.3. Spacecraft S' zips past Earth S at $u = 0.6c$. O-O' sets $t = t' = 0$ is when they pass. $\gamma = 1/\sqrt{1-\beta^2} = 1.25$

observer in S (stanley) sees S' (mavis) a short time later at a position they measure to be 9×10^7 m away.

stanley thinks this takes $\Delta t = \ell/u = 9 \times 10^7/(0.6 \times 3 \times 10^8) = 0.5\ s$

how long does Mavis think this takes? for mavis, these events happen at the same point! because at first she lines up with stanley, and then she lines up with the space station. so mavis measures the proper time, which is always the SHORTEST possible time interval. so she's going to measure a shorter time $\Delta t' = \Delta t/\gamma = 0.4$ s

stanley blinks as mavis flys past. Mavis measures the blink to take 0.4 s. how long does Stanley think this takes? its very tempting to say 0.5 s but this is NOT correct because its a DIFFERENT pair of events, and now its STANLEY which is at rest with respect to the two events (time taken for stanleys eyelid to go down and up).

so in effect you should relabel your frames. STANLEY is now in the frame S' in which the event is at rest. and MAVIS is in S which is moving relative to S' with velocity $-u$. but time dilation depends only on u^2 so direction doesn't matter. so then STANLEY is in S' so $\Delta t' = \Delta t/\gamma = 0.4/1.25 = 0.32$ s.

this gets us nicely back to our original discussion of simulteneity. Mavis passes the space station after 0.4 s, as stanley finishes his blink. Yet to stanley, he thinks his blink ends after 0.32 s, way before mavis passes the space station at 0.5 s. but we don't care, as these events are NOT causal.

1.5.3 twin paradox

But surely we can make this causal. take twins eartha and astrid. Astrid flys away from earth at high speed u, so all time intervals in her rest frame are shortest - including heartbeats! so she ages more slowly than eartha.

But since sign doesn't matter she can turn around and come back at the same speed and still age less. but then she can meet up with her twin and then astrid is younger than eartha.

But all inertial frames are relative so surely eartha could say the same - that she has gone away from astrid at $-u$ and then turned around and met up with her. so eartha should be younger!

but these frames are NOT symmetric - astrid has swapped inertial frames whereas eartha has always been in the same one (easier to think of it this way than to get tangled with acceleration/deceleration). so its not symmetric and astrid is indeed younger.

1.6 relativity of length

speed=distance/time. and we've seen that time goes funny because of the fixed speed of light. but length does as well. its back to the concept of simulteneity and how this differs for different observers. We measure can length of an object like a car by making marks on a stationary (relative to us) pavement at the front and back of the car and measuring between them. but if the car is moving then we have to make the marks simultaneously to get the true length of the car - if instead we marked the position of the back of the car a bit later than when we measured the front hen we could get -ve length as the back is now further forward than the front was when we measured it. but if length involves marking simultenously the front and back then we are back into the issue we had where simultaneous is a frame dependent concept (though causality isn't).

1.7 lengths parallel to motion

lets set up our standard frames S and S'. Put a ruler in S' (so its moving with velocity u wrt S) and attach a light to one end, and a mirror to the other. the total distance is go along the ruler and back to the same point is $2l'$ in the rest frame, and the time interval (proper time as its all in the rest frame of the ruler) between the light signal starting and being recieved is $\Delta t' = \Delta t'_1 + \Delta t'_2$ where $\Delta t'_1 = \Delta t'_2$ as the light trip is symmetric there and back in the ruler rest frame. speed=distance/time so $c = 2l'/\Delta t' = 2l'/\Delta t'$.

In frame S we know that the total length that the light has to travel is the length of the ruler in this frame, l plus the frame shift. The length to the mirror $d_1 = l + u\Delta t_1$ and it goes at the speed of light so $c = d_1/\Delta t_1$ so $c\Delta t_1 = l + u\Delta t_1$ and $c\Delta t_1 = l/(c-u)$

but on the way back we have $d_2 = l - u\Delta t_2$ and $c = d_2/\Delta t_2$ so $c\Delta t_2 = l - u\Delta t_2$ so $c\Delta t_2 = l/(c+u)$

The total time measured in S is $\Delta t = \Delta t_1 + \Delta t_2$

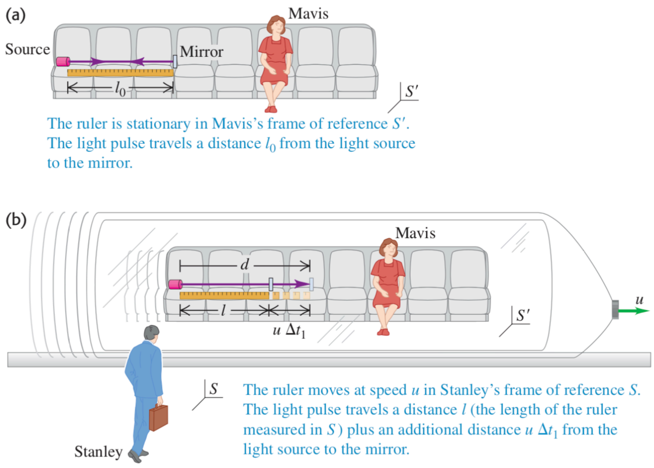

Figure 4:

$$\Delta t = \frac{l}{c-u} + \frac{l}{c+u} = \frac{2l/c}{1-u^2/c^2} = \gamma^2 2l/c$$

but we also know how the time intervals change from one frame to another $\Delta t = \gamma \Delta t'$

substitute in and get $\gamma \Delta t' = \gamma^2 2l/c$ so $\Delta t' = \gamma 2l/c$

but we had above that $\Delta t' = 2l'/c$ so we equate and so $2l'/c = \gamma 2l/c$ which gives us $l = l'/\gamma$.

This is length contraction. It is REAL not an optical illusion, in the same way that time dialation is REAL - we age less quickly if we move fast - fast clocks run slow.

when $u \ll c$ then $l \sim l'$ and we are back to classical mechanics. However, when $u \to c$ then $l \ll l'$

We call l' - length measured in the rest frame of the - PROPER DISTANCE - in the same way that time intervals measured at a single point are PROPER TIME

Example FY37.4 A spacecraft flies past the earth at a speed of 0.99c. A crew member on board the spacecraft measues its length to be 400m. What length do observers on earth measure?

S' is the frame of the spacecraft. $l' = 400$ m is proper length.

S is earth frame, so $l = l'/\gamma$ and $\gamma = 1/\sqrt{1-0.99^2} = 7.09$ so $l = 400/7.09 = 56.4$ m

lets keep going onto example YF37.5. Suppose 2 observers on earth are 56.4 m apart. how far apart does the spacecraft crew measure them to be?

This is a DIFFERENT EVENT so change frames. Now the EARTH is the rest frame so call it S', and the spacecraft is the new frame S that we want to consider. They still have relative velocity u so $gamma = 7.09$ again. But now we are looking at distance in S which is the SPACECRAFT frame relating to a proper distance l' measured in a rest frame so $l = l'/\gamma = 56.4/7.09$ so $l = 7.96$ m

THIS IS NOT THE PROPER LENGTH OF THE SPACECRAFT. As measured on earth it is the length of the spacecraft when the nose and tail are simultaneously measured. In the spacecraft frame these two positions are only 7.96m apart and the nose is 400m in front of the tail passes O2 before the tail passes O1.

1.7.1 lengths perpendicular to motion

actually we have already assumed that this didn't change in our discussion of the time transformations!!

and its easy to see this as the base of the ruler coincided with the x-x' origin. but it lies directly on the y-y' axis. so as the frame comes past at $t = t' = 0$ then it all lines up.