I didn't tell you what happened to v'_y and v'_z! so lets do this

$$x' = \gamma(x - ut) \quad y' = y \quad z' = z \quad t' = \gamma(t - xu/c^2)$$

but we can re-write them considering only a small displacement dx, dy, dz, dt and get

$$dx' = \gamma(dx - udt) \quad dy' = dy \quad dz' = dz \quad dt' = \gamma(dt - dxu/c^2)$$

$$v'_y = dy'/dt' = \frac{dy}{\gamma(dt - dxu/c^2)} = \frac{dy/dt}{\gamma(1 - dx/dtu/c^2)} = \frac{v_y}{\gamma(1 - v_x u/c^2)}$$

thats a bit unexpected!! $v'_y \neq v_y$! because even though $dy' = dy$, $dt' \neq dt$ - its the time change between frames which leads to a velocity change!

1.10 relativistic momentum

One of the key assumptions (postulates) was that physical laws are the same in all inertial frames. but our velocity transformations are very different to those in classical mechanics for speeds close to the speed of light, because there is now an ultimate speed limit. so accelerating a particle, giving it more and more energy, does NOT result in an increase in the (classically defined) KE... if we are going at 0.99c and then are accelerated to 0.9999c - we'd have pretty much the same momentum and KE if $p = mv \approx mc$ and $T = 1/2mv^2 = 1/2mc^2$...

yet we have poured energy in to accelerate the particle. so what has happened to physical laws of conservation of energy and momentum! how do we make these look the same in all inertial frames?

its a pain to derive, but you can set up some collisions and analyse them to see that the velocity transformations imply that momentum is not the classical mechanics $\vec{p} = m\vec{v}$ but is instead $\vec{p} = \gamma(v)m\vec{v}$ where m is proper (or rest) mass of a particle measured in its rest frame and $\gamma(v) = (1 - v^2/c^2)^{-1/2}$ and $v = |\vec{v}|$.

some justification as to why this might be the correct answer is to note that there is only one 'special' frame in which to measure time and thats the particle rest frame. time in all other frames is dialated (longer) so $dt = \gamma dT_0$ so maybe we really want $dx/dT_0 = dx/dtdt/dT_0 = \gamma v$

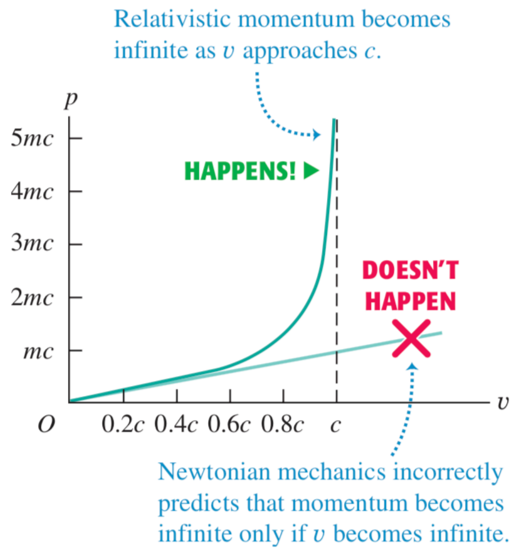

Figure 7:

Example: an oil tanker of mass 100kT is travelling at 0.3 m/s. how fast must a 1g hummingbird fly to have the same momentum

the tanker is going very slowly so we can surely use newtonian expressions $p_{tanker} \approx mv = 100 \times 10^6 \times 0.3 = 3 \times 10^7$ Ns

what about the bird - if we used newtonian we'd get $(mv)_{bird} = (mv)_{tanker}$ so $v_{bird} = v_{tanker} m_{tanker}/m_{bird} = 0.3 \times 10^8/10^{-3} = 3 \times 10^{10} = 100c$!!!

so we need to use the relativisitc expression $p_{bird} = \gamma(v)mv = m\beta c/\sqrt{1-\beta^2} = 3 \times 10^7$

$\beta/\sqrt{1-\beta^2} = 0.1/1e-3 = 100$ and $\beta^2 = 10^4/(10^4+1)$ so $\beta = 0.99995c$!!

So when DO we really need to use the proper relativisitic expression?

Example: at what speed does the newtonian expression give an error of 5%

difference between newtonian and relativistic is γ so when $\gamma = 1.05$ we get a 5% difference in momentum

$1/\sqrt{1-\beta^2} = 1.05$ so $\beta^2 = 1 - 1/1.05^2 = 0.0929$ so $\beta = 0.30c$

1.11 relativistic force

Now we have momentum defined properly we are good to go. We can get directly to force via the old classical mechanics $\vec{F} = d\vec{p}/dt = d/dt[\gamma(v)m\vec{v}]$

but surely special relativity is all about inertial frames?? so how can it handle acceleration? but we can always define an instantaneous inertial frame. an accelerating objects moves in continuous fashion from one inertial frame to another. so we can always define some instantaneous inertial frame S' which moves at velocity u with respect to another inertial frame S. Then in S we have an acceleration a for an objects whose instantaneous velocity is v while the same object in S' is accelerated by a' and has instantaneous velocity v'.

lets taks a couple of limiting cases. suppose $\vec{F}$ and $\vec{v}$ are both along the x-axis so the force is accelerating the particle along the direction of motion, increasing its velocity.

$$F_x = \frac{d\gamma}{dt}mv_x + \gamma m\frac{dv_x}{dt} = \frac{d}{dt}\frac{1}{(1-v_x^2/c^2)^{1/2}} + \gamma ma_x$$

we need to chain rule the 1st term.

$$\frac{d}{dt}\frac{1}{(1-v_x^2/c^2)^{1/2}} = \frac{d}{dt}(1-\beta^2)^{-1/2} = \frac{-1}{2}(1-\beta^2)^{-3/2}\frac{d(1-\beta^2)}{dt}$$

$$= \frac{-1}{2}(1-\beta^2)^{-3/2} - 2\beta\frac{d\beta}{dt} = (1-\beta^2)^{-3/2}\frac{v_x}{c}\frac{a_x}{c}$$

put this on the end and get

$$F_x = mv_x\gamma^3\frac{v_x}{c}\frac{a_x}{c} + \gamma ma_x$$

$$= \left(\gamma^2 v_x^2/c^2 + 1\right)\gamma ma_x = \frac{v_x^2/c^2 + (1 - v^2x^2/c^2)}{1 - v_x^2/c^2}\gamma ma_x = \gamma^3 ma_x$$

if instead $\vec{F}$ and $\vec{v}$ are perpendicular then we get something different. $\vec{F}$ acts perpendicular to $\vec{v}$ so causes the particle to go round in a circle rather than increasing its velocity. so v is constant in magnitude, so $d\gamma/dt = 0$. then we get $\vec{F} = \gamma m\vec{a}$.

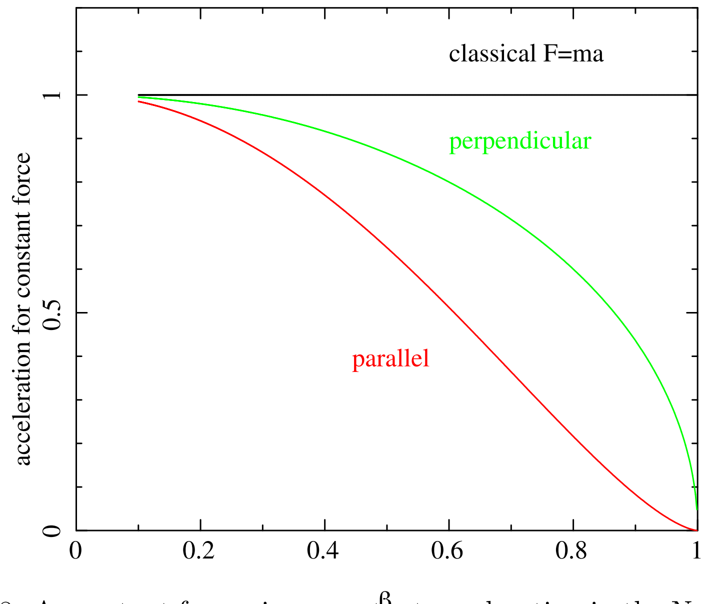

Figure 8: A constant force gives constant acceleration in the Newtonian limit as $F = ma$. But with relativity then if the force is parallel to the direction of motion you get the red line, where $a \propto \gamma^{-3}$ whereas if its perpendicular you get the green line, with $a \propto \gamma^{-1}$.

example YF37.9 An electron (mass 9.11e-31kg and charge -1.6e-19C) is moving opposte to an electric field of magnitude $E = 5 \times 10^5$ N/C. Find the magnitude of momentum and acceleration at the point when $v = 0.01c, 0.9c$ and $0.99c$.

$p = \gamma m v$ so the different v's imply different γ's of 1.0005, 2.29 and 7.09. hence these velocities have momenta $2.73 \times 10^{-23}, 5.64 \times 10^{-22}, 1.92 \times 10^{-21}$ kg/m/s

acceleration - this is acting in the same direction as the velocity (opposite but changing velocity so we have to consider the $d\gamma/dt$ term. so then we are using $F = \gamma^3 m\vec{a}$ and the force is given by the field $|F| = |q|E = 8 \times 10^{-14}$ N. Hence acceleration $|a| = F/(\gamma^3 m)$ so for each velcoity this is $|a| = 8.8 \times 10^{16}, 7.3 \times 10^{15}, 2.5 \times 10^{14}$ m/s/s so we can see that the same force does NOT give rise to the same acceleration - which is as expected as we can't go faster than c!

if instead this force had been perpendicular to the velocity we would see no change in speed, but there is an acceleration which changes direction. now its $|a| = F/(\gamma m)$ so its $8.8 \times 10^{16}, 3.8 \times 10^{16}, 1.2 \times 10^{16}$ m/s/s.