2.3 Compton effect

We did this in special relativity as an example of a massless particle collision! we had for our 1D collision $4pp' = 2(p - p')mc$, but if we'd done this in 2D we'd have got $2pp'(1 - \cos\phi) = 2(p - p')mc$. Use this to solve and get

$1 - \cos\phi = mc/p' - mc/p$

but we know that $p = h/\lambda$ so this is

$(1 - \cos\phi)h/mc = \lambda' - \lambda$

example. shine 0.124nm X-rays onto stationary electrons.

i) what angle is the wavelength of the scattered photons 1% longer than those incident?

$\lambda' = 1.01\lambda$ and $\lambda = 0.124$nm

$(1 - \cos\phi)h/mc = 0.01\lambda$

$1 - \cos\phi = 0.01\lambda mc/h$ so $\cos\phi = 1 - 0.010.124 \times 10^{-9} 9.1 \times 10^{31} 3 \times 10^8/6.626 \times 10^{-34} = 1 - 0.51 = 0.49$

$\phi = \cos^{-1}(0.49) = 60.6°$.

ii) what angle is the maximum wavelength change? and what is this maximum change in nm?

$\lambda' - \lambda = h/mc(1 - -1) = 2h/mc = 0.0048$ nm - for incident $\lambda = 0.124$nm then this is not quite a 4% change but if we went to shorter wavelengths - higher energies - we can make it a bigger fractional change but the compton wavelength itself is fixed!

$\cos\phi = -1$ so $\phi = 180$ (head on collision, like the one we did)

iii) what angle is the minimum wavelength change - and what is this?

$\lambda' - \lambda = h/mc(1 - 1) = 0$

$\cos\phi = 1$ so 0 degrees - grazing.

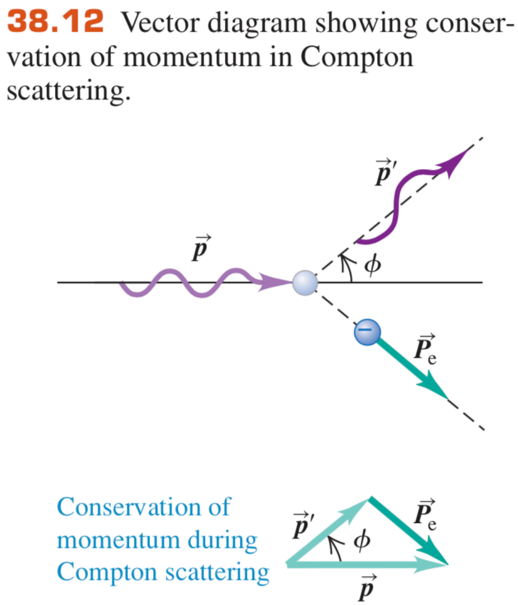


Figure 1:

2.3.1 pair production

another effect which can be explained only in the photon picture involves high energy gammma rays. If a gamma ray with sufficiently short wavelength is fired at a target then it need not scatter, instead, it can disappear completely and be replaced by an electron positron pair.

so it must have energy $E > 2m_ec^2 = 2 \times 0.51 MeV/c^2c^2 = 1.022$ MeV

or in standard SI units $2m_ec^2 = 2 \times 9.1 \times 10^{-31} \times 9 \times 10^{16} = 1.637 \times 10^{-13}$ J

$E = hc/\lambda$ so needs $\lambda < hc/E_min = 6.6 \times 10^{-34} 3 \times 10^8 / 1.637 \times 10^{-13} = 1.213 \times 10^{-12}$ m

similarly an electron positron pair can annihilate into TWO photons - (need to conserve energy and momentum!)

its easiest in the centre of mass/energy frame

$2\gamma mc^2 = 2E$

for $\gamma \sim 1$ is KE to zero we get

$mc^2 = E$ and E=0.51MeV. $hf = E/h = 1.24 \times 10^{20}$ Hz

2.4 wave-particle duality

so electromagnetic waves can look like particles (photons)

but they do also look like waves - we saw they did interference patterns in youngs double slit experiment.

lets turn down the intensity of the beam so that only 1 photon on average goes through at any one time. then it will have to pick only one slit, and we'll just see the 2 slit pattern on the screen, not the full interference pattern....but thats not what is seen! we do indeed see individual photons, but after a while the full interference pattern builds up NOT the 2 slits!

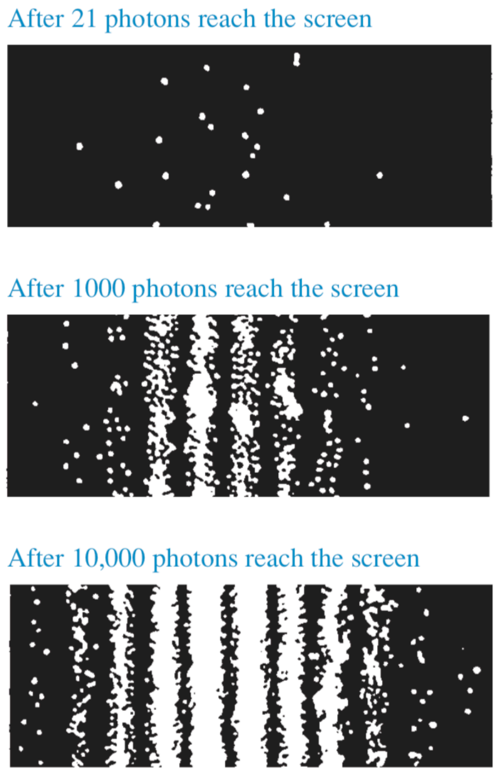
so there is no classical description that works - we can't have just particles (photons to explain photoelectric effect) or just waves (to explain interference). We somehow need both. this is wave particle duality.

no classical description in terms of just particles or just waves works

the 'standard' way to think about this involves some double-think! at this level you will never need to simultaneously think of them being both particles and waves. this 'principle of complementarity' is that we need both, but not at the same time!

back to double slit, with only 1 photon at a time. we record the discrete photons as single events on a CCD detector. we cannot predict where any

38.16 These images record the positions where individual photons in a two-slit interference experiment strike the screen. As more photons reach the screen, a recognizable interference pattern appears.



given photon will strike, but over time they make the diffraction patteren we know and love....

to recocile particle and wave aspects we regard the pattern as a statistical distribution. the interference pattern gives us a probability distribution for any given photon to land at any spot.

the wave description gives us the interference, the particle description gives us the individual events on the screen that we detect with the CCD

particle aspect becomes apparent at the moment of detection.

there is no such thing as a specific trajectory (which slit did the photon go through??)

photon arrival is unpredictable. fringe pattern is probability.

light is associated with a wave such that probability of finding photon is proportional to (amplidute of wave)^2.

$\vec{E} = \vec{E_0} \sin[2\pi(x/\lambda - ft)]$ and $I \propto \vec{E}^2$

the wave amplitude contains ALL THAT CAN BE KNOWN (we can't know phase)

2.4.1 probability and uncertainty

a single slit we get diffration of a wave - we know its position well at the slit, but the diffraction means that its momentum can be over a very large range! we get the width $\sin\theta_1 = \lambda/a$.

If $\lambda \ll a$ then we use small angle approximation to get the ampint of spread on the screen. The diffraction angle $\sin\theta_1 \approx \theta_1 = \lambda/a$

so there is a y component of momentum - which must be balanced, so this is actually an uncertainty in $\pm p_y$ of $\Delta p_y/p_x = \tan\theta_1 \approx \theta_1$

$\Delta p_y = p_x\theta_1 = p_x\lambda/a$

so the narrower the slit, the broader the diffraction pattern and the greater the uncertainty in y-component of momentum!

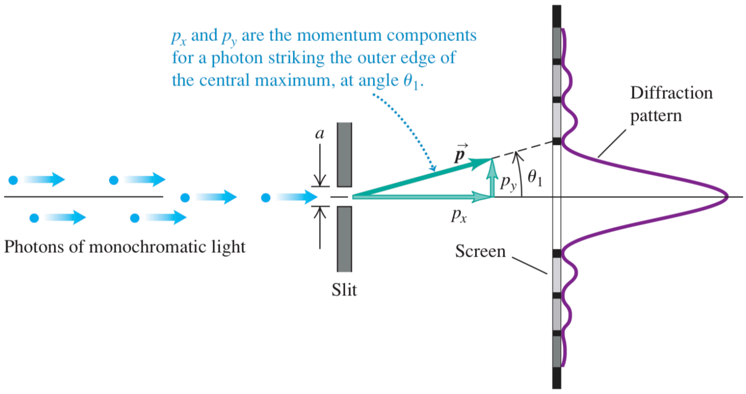
but we said that light has mometum $p_x = hf/c = h/\lambda$ so

$\Delta p_y = h/\lambda\lambda/a = h/a$

but a represents the uncertainty in its y position - it has to go through the slit! so $\Delta y = \pm a/2$ or $2\Delta y = a$

so then $\Delta p_y = h/(2\Delta y)$ or $\Delta p_y \Delta y = h/2$

the narrower the slit, the better we know its y position, but the broader the diffraction pattern, and the greater the uncertainty in the p_y momentum.



2.4.2 Heisenburg uncertainty principle

we can do this in any direction, and when we do this more carefully there is an extra factor of 2π so $\Delta x \Delta p_x \geq \hbar/2$ where $\hbar = h/2\pi = 1.05 \times 10^{34}$ Js.

we cannot evade this - eg if we try to measure the position of a particle by using a photon. a short wavelength allows for a precise location but the compton scattering significantly changes the particle momentum. Instead use a long wavelength photon and the opposite is true.

but actually its more fundamental than just how you measure!

Example: bullet $m = 0.01$ kg and electron $m = 9.1 \times 10^{-31}$ kg have velocity of 300 m/s measured with uncertaity of 0.01% what is the funamental accuracy of a simultaneous measurement of position!!

$p = mv$ and $\Delta v/v = 10^{-4}$ so $\Delta p/p = \Delta v/v = 10^{-4}$ and $\Delta p = 10^{-4} p$

$\Delta x = \hbar/\Delta p = 10^{-34}/(10^{-4}p) = 10^{-30}/p$

so for the bullet is $p = 3kg/m/s$ so $\Delta x \sim 3 \times 10^{-31}$ m which is a LOT smaller than the size of the bullet@ so this isn't going to be important!

for the electron its $p = 9.1 \times 10^{-31} 300 = 2.7 \times 10^{-28}$ kg/m/s so $\Delta x \sim$ 0.004 m

2.4.3 wavepacket localisation

eg wavepackets. we have a wave of single frequency and we know well its momentum.

$E_y(x,t) = A\sin(kx - \omega t)$ where $k = 2\pi/\lambda$

$p_x = h/\lambda = (h/2\pi)(2\pi/\lambda) = \hbar k$

we know p_x exactly but we know nothing about its position as its over all space!

we can localise it by interference with different wavelengths. the more wavelengths we add into the pattern, the tighter the localisation of the sum. but the larger the range of momenta as $p = h/\lambda$.

this makes it clear that there is another uncertainty principle in terms of energy and time

2.4.4 Energy-time uncertainty!

so since we have $\Delta p_x \Delta x \geq \hbar/2$ then we expect an energy-time uncertainty principle $\Delta E \Delta t \geq \hbar/2$.

travelling wavepacket - moves in time Δt past a given point.

standing waves - interference - so amplitude can be zero when the waves exactly cancel for a short time before they exactly add! so see zero energy or 2x energy on time $\Delta t \sim 1/f = h/E$.

Example: prediction of pions!

We know the strong nuclear force has a range of $R = 1.4 \times 10^{-15}$ m if this is carried by particles then $\Delta t = R/c = 4.66 \times 10^{-24}$ s so $\Delta E = \hbar/(2\Delta t) = 1.13 \times 10^{-11}$ J

$E = mc^2$ so $m = 2.25 \times 10^{-28}$ kg

which is actually pretty accurate!!