aside: Pole in a barn 'paradox' in special relativity

This is a classic braintwister, up there with the twins.

A pole is 20m long, a barn is 10m long, but the pole is moving at u=0.9c relative to the barn. In the barn frame this is length contracted to 8.8m so it easily fits in, but in the pole frame the BARN is contracted to 4.4m while the pole is still 20m so there is no way for it to fit!!!

We can't just say 'point of view' if we slam the doors on the pole - in the barn frame the doors can slam and open with the pole in the barn so there is no collision, but if the pole DOESN'T fit in the barn then there is a collision! Physics (whether or not there is a collision) HAS to be the same in all frames. so what happens??

This is where simulteneity comes in. The doors close SIMULTANEOUSLY in the barn frame. But the doors are at different x positions. so what does simultaneous mean ?? The standard Lorentz transformations tell us that

$t' = \gamma(t - ux/c^2)$ so $\Delta t' = \gamma(\Delta t - u\Delta x/c^2)$

for two events, at x1 and t1, and x2 and t2, we can form a difference so that $\Delta t' = \gamma(\Delta t - u\Delta x/c^2)$.

if things are simultaneous in the barn frame, then $\Delta t = 0$. but then this is only simultaneous in another frame (like the pole runners!) if $\Delta x = 0$ or $u = 0$. So if there is no relative motion then times agree (as in Newtonian mechanics) but if there is relative motion then 'simultaneous' only works if $\Delta x = 0$ i.e. the events occur at the same place as well as the same time! Incidentally, this also tells you that our time dilation (and length contraction) formulae are a bit sloppy. its always MUCH safer to use the lorentz transforms and transform space and time coordinates together.

$t' = \gamma(t - ux/c^2)$ so multiply by c and get a more symmetric looking equation $ct' = \gamma(ct - \beta x)$

and $x' = \gamma(x - ut)$ which is equivalent to $x' = \gamma(x - \beta ct)$

we can use these to construct spacetime diagrams to get a bit more insight

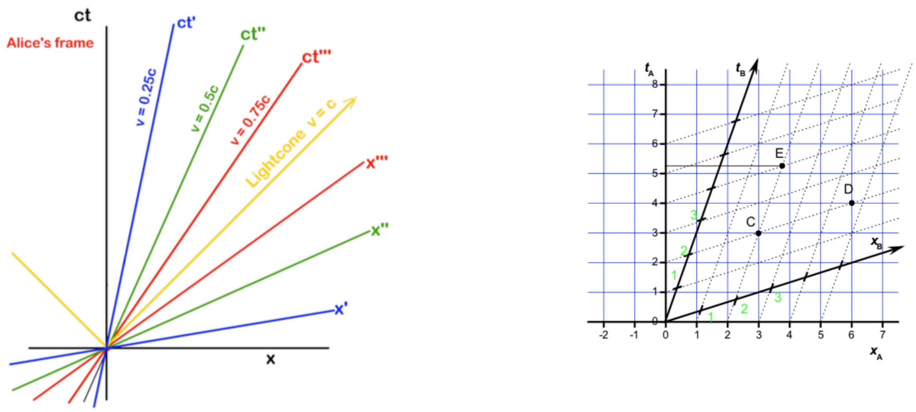

Figure 1: a) spacetime axes for increasing β: b, simultaneiy depends on frame!

into what is going on. Normally we show x-y axes, but on a spacetime diagram we supress y as its not changing in anything we do, and replace it with time instead. so stationary observers are a vertical line in a spacetime diagram.

So now lets shift into another frame, and do the standard synchronisation $x = x' = t = t' = 0$ at the vertex.

x' axis has $t' = 0$ so this starts at $ct = ct' = 0$ and has $ct' = 0$ so its given by $0 = \gamma(ct - \beta x)$ i.e. $ct = \beta x$

the t' axis starts at $x = x' = 0$ and then has $x' = 0$ so its given by $0 = \gamma(x - \beta ct$ so its equation is $ct = x/\beta$.

so now we can draw this and see that the angle between x and x' and the angle between t and t' is given by $\tan\theta = \beta$.

so as we get closer and closer to the speed of light, we squeeze our axes closer and closer to the light cone.

This gives a nice way to visualise simulteneity - just dot on a line of constant

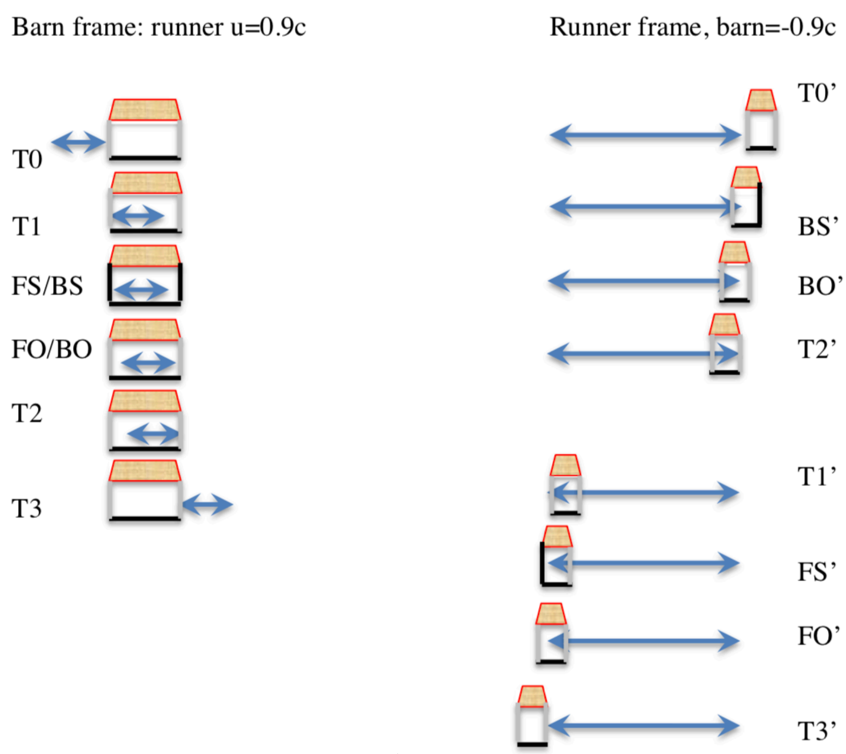

Figure 2: Barn frame view versus runner/pole frame view of events

ct and another line of constant ct' to see that they are different!!

so what does this do for our pole in barn - it means the order of events changes. in the runner frame the pole enters the front of the barn, then the door at the back of the barn shuts, then it opens, then the pole gets to the back of the barn but the door is already open so there is no collision. And then the back of the pole gets into the front of the barn, then the front door shuts but the pole is already inside so there is no collision here either!