ELEMENTS OF QUANTUM MECHANICS - EXAMPLES CLASS 2

Q1 The infinite potential well in 1 dimension has energy eigenfunctions ψ_n with energy $E_n = n^2 E_1$. An electron in this potential has a wavefunction
$\psi(x, t = 0) = A[\psi_1 + 3\psi_2 + \psi_3]$.

(a) Use the normalisation condition on $\psi(x, t = 0)$ and the orthonormal properties of the energy eigenfunctions to show that $A = 1/\sqrt{11}$.

(b) What is the probability (3 sig figs) that a measurement of energy gives E_1?

(c) If the system has been measured to have energy E_1, what is the probability that a subsequent measurement of energy will give E_2?

(d) Write down $< E >$ in terms of E_1 and calculate it. can any single measurement of E give this value?

Q2. An electron in the infinite potential well has a wavefunction which is symmetric about its midpoint. What is the probability that the system is found in state ψ_2? Hint: draw the n=1,2 and 3 wavefunctions of the infinite square well, and consider their symmetry properties.

Q3. Sketch (without calculation) the potential and its wavefunctions of the n=1,2 and 3 levels. For each potential, comment on whether the number of bound states is finite or infinite.

(a) the finite square well potential where $V = 0$ for $-L/2 < x < L/2$ and $V = V_0$ elsewhere.

(b) the inverse gaussian potential $V = V_0(1 - e^{-x^2/2L^2})$ for $-\infty < x < \infty$

Q4. For the finite square well potential in 3a, write down the Schroedinger equation and solve it for the region $x > L/2$. Simplify your solution by considering the behaviour of the wavefunction at ∞. Is the exponential decay constant the same for each of $n = 1, 2$ and 3? Repeat your sketch from 3a including this information. Give a physical reason why the $n = 3$ electron can penetrate further into the classically forbidden region.