ELEMENTS OF QUANTUM MECHANICS - EXAMPLES CLASS 2

Q1 $\psi(x,t=0) = A[\psi_1 + 3\psi_2 + \psi_3]$.

(a) $\int \psi(x,t=0)^*\psi(x,t=0)dx = 1$
$= A^2 \int(\psi_1 + 3\psi_2 + \psi_3)^*(\psi_1 + 3\psi_2 + \psi_3)dx$
$= A^2 \int(\psi_1^* + 3\psi_2^* + \psi_3^*)(\psi_1 + 3\psi_2 + \psi_3)dx$

$$= A^2[\int \psi_1^*\psi_1 + \psi_1^* 3\psi_2 + \psi_1^*\psi_3 + 3\psi_2^*\psi_1 + 9\psi_2^*\psi_2 + 3\psi_2^*\psi_3 + \psi_3^*\psi_1 + \psi_3^* 3\psi_2 + \psi_3^*\psi_3 dx]$$

$= A^2(\delta_{11} + 3\delta_{12} + \delta_{13} + 3\delta_{21} + 9\delta_{22} + 3\delta_{23} + \delta_{31} + 3\delta_{32} + \delta_{33})$
$= A^2(11)$ hence $A = 1/\sqrt{11}$

(b) prob measure E_1 is $|c_1|^2$ ie $1/11 = 0.0909$

(c) zero! the system is now in a pure state ψ_1 which has energy E_1 so there is no probability to measure energy E_2

(d) $< E >= c_1^2E_1 + c_2^2E_2 + c_3^2E_3 = 1/11E_1 + 9/11E_2 + 1/11E_3$
$= (1/11 + 9/11 \times 4 + 1/11 \times 9)E_1 = 46/11E_1 = 4.18E_1$
can't get this from a single measurement as only energies which you can get are E_1, $4E_1$ and $9E_1$

Q2. zero! ψ_2 is antisymmetric about the midpoint! so no wavefunction which is overall symmetic about the midpoint can have any contribution from this component. see sketch.

Q3. see sketch

(a) finite number of bound states

(d) infinite number of bound states

Q4

$$\frac{-\hbar^2}{2m}\frac{\partial\psi^2}{\partial x^2} = \frac{2m}{\hbar^2}(V_0 - E)\psi = \rho^2\psi$$

general solution $\psi = Ce^{\rho x} + De^{-\rho x}$
boundary condition at $x \to \infty$ is $C = 0$ so $\psi = De^{-\rho x}$.
$\rho \propto \sqrt{(V_0 - E)}$ so gets SMALLER as E gets bigger, approaching V_0. so the exponential tail gets LONGER. The particle can penetrate further into the forbidden region when the potential barrier is smaller. so $n = 3$ has the longest tail into the classically forbidden region, and $n = 1$ has the smallest - see my sketch answer to 3a.