ELEMENTS OF QUANTUM MECHANICS - EXAMPLES CLASS 5
Answers

Q1 The electron in a Hydrogen atom is in a combined state

$$\psi(r,\theta,\phi) = \sqrt{\frac{1}{21}}(2\psi_{210} + \psi_{321} + 4\psi_{422})$$

(a) L^2 has eigenvalues $l(l+1)\hbar^2$ so we get values of $2\hbar^2$ for $l = 1$ with probability $4/21$ and value $6\hbar^2$ for $l = 2$ with probability $17/21$ (as there are two with $l = 2$)

(b) L_z has eigenvalues $m\hbar$ so $m = 0$ we get 0 for $4/21$ and $m = 1$ we get $\hbar$ for $1/21$ and $m = 2$ we get $2\hbar$ for $16/21$

(c) H i.e. energy is E_2 for $4/21$, E_3 for $1/21$ and E_4 for $16/21$

(d) if measure energy $-13.6/9$ eV it means that n=3 so we are looking at ψ_{321}. A subsequent measure of E is certain to give the same value (collapse of the wavefunction). and measuring L^2 is certain to give $6\hbar^2$ ($l = 2$) and L_z is certain to give $\hbar$. All of these can be measured simultaneously, without disturbing the wavefunction as they all commute. BUT if we now try and measure L_x this does not commute so we don't get a deterministic answer.

Q2

(a) $E_3 - E_2 = -13.6(1/3^2 - 1/2^2) = 1.89$ eV
$E_4 - E - 2 = -13.6(1/16 - 1/4) = 2.55$ eV

(b) $mu_D = 2m_pm_e/(2m_p + m_e) = 2m_p/(2m_p/m_e + 1)$
$\mu_H = m_pm_e/(m_p + m_e)$
$\mu_D/\mu_H = 2(m_p + m_e)/(2m_p + m_e) = 2(m_p/m_e + 1)/(2m_p/m_e + 1) = 1.000272$
$a_D/a_H = \mu_H/\mu_D$
$E_{n,D}/E_{n,H} = (\mu_H/\mu_D)(a_H^2/a_D^2) = \mu_D/\mu_H = 1.000272$
$(E_{n,D} - E_{n,H}) = 1.000272E_{n,H} - E_{n,H} = 0.000272E_{n,H}$
so the shift due to the presence of some deutreium is 5×10^{-4} eV, which is WAY bigger than observed!

Q3 $p = -i\hbar d/dx$
$[H,p]\psi = [p^2/2m, p]\psi + [V,p]\psi = [V,p]\psi$
$[H,p]\psi = [V,p]\psi = Vp\psi - p(V\psi) = V. - i\hbar\frac{d\psi}{dx} - -i\hbar d/dx(V\psi)$
$= -i\hbar(Vd\psi/dx - Vd\psi/dx - \frac{dV}{dx}\psi) = i\hbar\frac{dV}{dx}\psi$