PHYS2581 Foundations2A: examples class 8

Q1. A particle with spin $s = 1$ is in an orbit which has orbital angular momentum quantum number $l = 2$. Total angular momentum is $\underline{J} = \underline{L} + \underline{S}$, forming operators J^2 and J_z, with eigenvalues $j(j+1)\hbar^2$ and $m_j\hbar$. What are the possible values resulting from measurements of J^2 and J_z. [Hint: do it systematically!! j runs from $|l - s|$ to $(l + s)$ in integer steps, and for each possible j give the possible values of m_j.]

What is the probability to measure $J_z = 0$?

Q2: The Lyman alpha line in Hydrogen involves transitions between $n = 2$ to $n = 1$. The energy of each level depends on n and j only.

(a) How many different line energies are there?

(b) Is the $n = 1$ level still degenerate after doing the fine structure corrections?

(c) If there were no other factors operating, what is the probability to get each line energy?

(d) Emission of a photon leads to a change in angular momentum $\Delta l = 1$ since the photon carries this away. How does this extra condition change the probability for each line energy?

Q3: We found first order corrections to the energy levels in Hydrogen from relativistic corrections, spin-orbit coupling and the finite size of the proton:

$$E^1_{n,r} = -\frac{(E^0_n)^2}{2mc^2}\Big[\frac{4n}{l+1/2} - 3\Big] \quad \text{which holds for all } l$$

$$E^1_{n,so} = \frac{(E^0_n)^2}{mc^2}\frac{n\Big(j(j+1) - l(l+1) - 3/4\Big)}{l(l+1/2)(l+1)} \quad l \neq 0$$

$$E^1_{n,0} = 2n\frac{(E^0_n)^2}{mc^2} \quad l = 0$$

add these together for the three separate cases $l \neq 0$ and $j = l - 1/2$, $l \neq 0$ and $j = l + 1/2$, $l = 0$ so $j = 1/2$