PHYS2581 Foundations2A: examples class8

Q1 j=—l-s—...l+s=1,2,3
j=1, m_j=-1,0, 1
j=2, m_j=-2,-1,0,1,2
j=3 m_j=-3,-2,-1,0,1,2,3

prob get $m_j = 0$ is 3/15=1/5

Q2: $n = 2$ can have l=0 or 1, s=1/2 so j=1/2 or 3/2
$n = 1$ can only have $l = 0$ so j=1/2

so there are two different line energies. To work out probabilities first look at the old ways - there are 8 for n=2, and 2 for n=1

n l m m_s
2 1 1 ±1/2
2 1 0 ±1/2
2 1 -1 ±1/2
2 0 0 ±1/2

n=2 l=1 can have j=3/2 so m_j=-3/2 -1/2 1/2 3/2
n=2 l=1 can have j=1/2 so m_j= -1/2 1/2
n=2 l=0 can have j=1/2 so m_j= -1/2 1/2
so it adds up to 8 for n=2 as required

n=1 l=0 can have j=1/2 so m_j= -1/2 1/2
so we have the correct number of states.

The line energies only depend on n and j so we have only 2 lines, with $n = 2, j = 3/2$ to $n = 1, j = 1/2$ or $n = 2, j = 1/2$ to $n = 1, j = 1/2$

so both states in $n = 1$ which come from $m_s = \pm 1/2$ are shifted by the same amount, so the level is still degenerate!

probability without anything else is 1/2 as 4/8 are in $n = 2, j = 3.2$ and 4/8 are in $n = -2, j = 1/2$.

BUT if we make it go from $n = 2, l = 1$ to $n = 1, l = 0$ then there still 2 lines but now we are only allowed to consider 6 of the 8 states in n=2 so its 4/6=2/3 for $n = 2, j = 1/2$ and 2/6=1/3 for $n = -2, j = 1/2$.

Q3

(a) $l \neq 0$ and $j = l + 1/2$ hence $l = j - 1/2$

$$E^1_{n,r} = -\frac{(E^0_n)^2}{2mc^2}\Big[\frac{4n}{l+1/2} - 3\Big] = -\frac{(E^0_n)^2}{2mc^2}\Big[\frac{4n}{j} - 3\Big]$$

$$E^1_{n,so} = \frac{(E^0_n)^2}{mc^2}\frac{n\Big(j(j+1) - (j-1/2)(j+1/2) - 3/4\Big)}{(j-1/2)j(j+1/2)}$$

$$= \frac{(E^0_n)^2}{mc^2}\frac{n\Big(j^2 + j - (j^2 - 1/4) - 3/4\Big)}{(j-1/2)j(j+1/2)}$$

$$= \frac{(E^0_n)^2}{mc^2}\frac{n(j-1/2)}{(j-1/2)j(j+1/2)} = \frac{(E^0_n)^2}{mc^2}\frac{n}{j(j+1/2)}$$

no term from $E^1_{n,0}$ so total is

$$E^1_n = -\frac{(E^0_n)^2}{2mc^2}\Big[\frac{4n}{j} - 3\Big] + \frac{(E^0_n)^2}{mc^2}\frac{n}{j(j+1/2)}$$

$$= -\frac{(E^0_n)^2}{2mc^2}\Big(\frac{4n}{j} - 3 - \frac{2n}{j(j+1/2)}\Big)$$

$$= \frac{(E^0_n)^2}{2mc^2}\Big(3 - \frac{4n(j+1/2) - 2n}{j(j+1/2)}\Big)$$

$$= \frac{(E^0_n)^2}{2mc^2}\Big(3 - \frac{4n(j+1/2)}{j(j+1/2)}\Big) = \frac{(E^0_n)^2}{2mc^2}\Big(3 - \frac{4n}{j+1/2}\Big)$$

(b) $l \neq 0$ and $j = l - 1/2$ i.e. $l = j + 1/2$

$$E^1_{n,r} = -\frac{(E^0_n)^2}{2mc^2}\Big[\frac{4n}{j+1} - 3\Big]$$

$$E^1_{n,so} = \frac{(E^0_n)^2}{mc^2}\frac{n\Big(j(j+1) - (j+1/2)(j+3/2) - 3/4\Big)}{(j+1/2)(j+1)(j+3/2)}$$

$$= \frac{(E^0_n)^2}{mc^2}\frac{-n}{(j+1/2)(j+1)}$$

$$E^1 = -\frac{(E^0_n)^2}{2mc^2}\Big[\frac{4n}{j+1} - 3\Big] - \frac{(E^0_n)^2}{2mc^2}\frac{-2n}{(j+1/2)(j+1)}$$

$$= \frac{(E^0_n)^2}{2mc^2}\Big[3 - \frac{4n(j+1/2) - 2n}{(j+1/2)(j+1)}\Big] = \frac{(E^0_n)^2}{2mc^2}\Big[3 - \frac{4n}{j+1/2}\Big]$$

(c) j runs from $|l - 1/2|$ to $l + 1/2$ so there is only a single j value of $j = 1/2$ if $l = 0$ so

$$E^1 = E^1_{n,r} + E^1_{n,0} = -\frac{(E^0_n)^2}{2mc^2}\Big[4n - 3\Big] + 2n\frac{(E^0_n)^2}{mc^2} = \frac{(E^0_n)^2}{2mc^2}\Big[3 - 4n\Big]$$

which is also consistent with the form in (a) since $j = 1/2$