(a) An electron in the 1D infinite potential square well where $V = 0$ from $0 < x < L$ has wavefunction $\Psi(x, t = 0) = \sqrt{2/L} \sin(\pi x/L)$. Write down the appropriate form of the momentum operator, p, and calculate $\langle p \rangle$. Comment on the physical significance of your answer. [4 marks]

(b) A 1D double finite square well potential has $V = 0$ for $L < x < 2L$ and $-2L < x < -L$, and V_0 elsewhere. The potential is large enough that several bound states occur. Sketch the potential and sketch the ground state and first excited state wavefunctions. Does the ground state wavefunction peak at the midpoint of each well, or is the peak displaced towards the origin or towards the outer edge of each well? Give a physical explanation for your answer. [4 marks]

(c) The 3D anisotropic harmonic oscillator has energies $E = \hbar\omega_x(n_x+1/2)+\hbar\omega_y(n_y+1/2) + \hbar\omega_z(n_z + 1/2)$ Write down the energy and degeneracy of the ground state and first excited state if $\omega_y = \omega_x$ and $\omega_z = 2\omega_x$. Give a qualitative description of the values of ω_x, ω_y and ω_z which maximize the degeneracy of the first excited state. What values minimise the degeneracy? [4 marks]

(d) The ground state wavefunction of Hydrogen is $\psi_{000}(r, \theta, \phi) = (\pi a^3)^{-1/2} e^{-r/a}$. Write down the probability to find the electron within a volume dV of position r, θ, ϕ. What is the radial probability distribution function $D(r)$, where $D(r)dr$ is the probability to find the electron within distance dr of r. Where does this probability distribution peak? [4 marks]

Q2: The 1D infinite square well potential, where $V(x) = 0$ for $0 < x < L$ and ∞ elsewhere, has energy eigenfunctions $\psi_n(x) = \sqrt{2/L} \sin n\pi x/L$, corresponding to energy $E_n = n^2\pi^2\hbar^2/(2mL^2) = n^2E_1$.

An electron in this potential has $\Psi(x, t = 0) = Ax(L - x)$ where $A = \sqrt{30/L^5}$. This can be expanded into an infinite sum over the energy eigenfunctions as $\Psi(x, t = 0) = \sum_n c_n\psi_n$ where $c_n = \frac{4\sqrt{15}}{n^3\pi^3}(1 - \cos(n\pi))$.

(a) Write down the first 3 terms explicitly, and use these to give a general form for odd n. What is the general form for even n? Give a physical explanation for your answer. [6 marks]

(b) Write down an infinite sum for $< E >$ in terms of E_1, and calculate this given that $\sum_{oddn} \frac{1}{n^4} = \frac{\pi^4}{96}$. Can any measurement of E give this value? [4 marks]