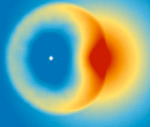


Implementation of feedback in SPH: towards concordance of methods



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Abstract

We perform simulations of feedback from supernovae with smoothed particle hydrodynamics (SPH). We show for the first time that, in the absence of radiative cooling, **concordance of thermal and kinetic feedback** is achieved when using an appropriate time integration. In order to preserve a high level of energy conservation when using the hierarchical time-step scheme, we implemented in the GADGET-2 code a modified version of the time-step limiter proposed by *Saitoh & Makino* (2009). We apply the limiter to general test cases, and find necessary, not only to ensure a **fast information propagation**, but also to enforce a **prompt response** of the system to the energy perturbation. The method proposed here to handle strong feedback events enables us to achieve energy conservation at percent level in all tests, even if all the available energy is injected into only one particle. Finally, we show that, even if cooling processes are taken into account and providing a sufficiently high resolution, simulations of an individual supernovae explosion with the different feedback methods are still reaching concordance.

Animations available at: <http://www.mpe.mpg.de/~fdurier/Concordance/>

Sedov's test

Results are shown for different time integration schemes. Upper panels show the projected density field in a slice (where the white dots correspond to the particles that initially received the energy and the dashed circle give the position of the expected blast radius), while lower panels compare the simulated density profiles (black) with the expected profile from the similarity solution at that time (red). For each test both thermal (left) and kinetic (right) feedback approaches are compared.

In the case of the standard **individual** time-stepping scheme, the information about the sudden injection of energy is not transferred from neighbouring particles. This leads to an extreme violation of energy conservation that produces inter-particle crossing for both feedback methods.

When the time-step **limiter** proposed by *Saitoh & Makino* is applied, thermal feedback produces a stable shell that develops too quickly ahead from the similarity solution. In the kinetic case, kicked particles are still able to travel to very large distances before interacting with the medium. In both cases, the delayed response of the medium after the explosion explains again the non-conservation of the input energy.

If an additional time-step **update** is enforced at the explosion time, both feedback methods give concordant results: the shell position and the radial density profile are in extremely good agreement with the analytic solution.

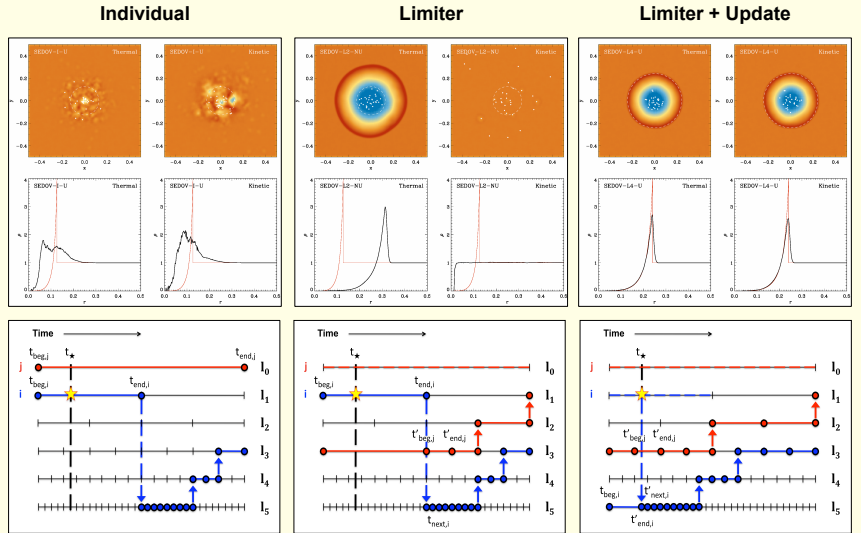
Time Integration Schemes

Energy is given at time t_* to particle i , and particle j is one of its neighbours (labels given on the right side of each sketch show the levels of the time-bin hierarchy).

- **Individual**: particles are only able to adapt their time-step when they become active. Therefore, heated/kicked particles may complete a significant number of steps before their neighbours become active as well.
- **Limiter**: neighbouring particles communicate to each other the length of their time-steps, and keep the ratio of long over short steps no larger than a fiducial factor of $f_{\text{step}} = 4$. However, particles still need to be active before their neighbours adjust their time-step accordingly to the limiter criterion.
- **Limiter + Update**: here we make sure that heated/kicked particles become active at the time of energy injection, and adjust their time-step accordingly to the amount of energy they receive. When applied in combination with time-step limiter, impacted particles and their neighbours can promptly react to the change of energy in the medium.

Qualitative Overview

Integration scheme	Energy conservation [%]		Computational time [min]	
	Thermal	Kinetic	Thermal	Kinetic
Global	0.93	1.35	139	101
Limiter only	4×10^4	43.46	119	35.4
Limiter + Update	3.03	2.86	21.8	18.2



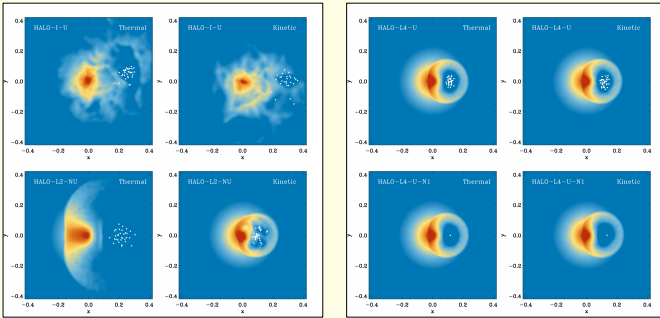
Halo's test

We present below the results of an off-centre explosion in a self-gravitating gas sphere using the same integration schemes as before. Both thermal and kinetic energy injection methods are considered.

- **Individual** (top-left): the behaviour is similarly wrong in both cases. The halo atmosphere is disrupted and no expanding bubble form.
- **Limiter** (bottom-left): the energy violation is severe in the thermal case and the bubble have blown away a large fraction of the gas halo.
- **Limiter + Update** (right): the results are qualitatively identical, even if all the available energy is injected into only one particle (bottom panels), showing again the concordance of the two feedback methods.

Individual / Limiter

Limiter + Update



Qualitative Overview

Integration scheme	Energy conservation [%]		Computational time [h]	
	Thermal	Kinetic	Thermal	Kinetic
Global	1.62	1.84	25.70	25.87
Individual	6×10^4	2×10^5	14.20	19.73
Limiter	3×10^3	19.42	6.07	3.73
Limiter + Update	2.16	2.60	3.41	3.39

What about Cooling?

Simulations of a single SN explosion including (orange lines) or not (black lines) cooling processes are compared in the figure below for both feedback methods. From top to bottom we see the evolution, of the thermal (solid) and kinetic (dash) budget, of the blast radius and of both the bubble (dash) and shell (solid) temperatures. Red lines give the expected Sedov radius and shell temperature as a reference.

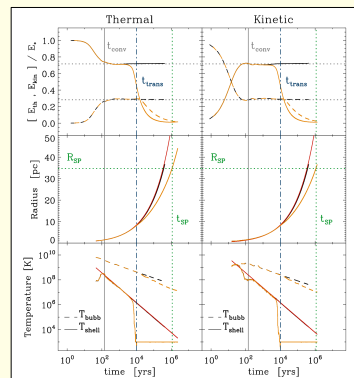
Firstly, we see that simulations need a certain amount of time to numerically converge to the Sedov phase (the expected energy partition is given by the grey horizontal dotted lines). This **convergence time** can be expressed as a function of the physical properties of the problem and of the numerical resolution as follow:

$$t_{\text{conv}} \approx 1.4 \times 10^4 E_{\star}^{1/2} n_0^{-2/3} m_{\star}^{3/8} \approx 140 \text{ [yrs]}$$

Secondly, following *Cox-1972* and *Blondin et al.-1998* we define the transition from the Sedov to the snowplough phases by the time at which the cooling time of the shell equals the blast age. For our choice of the cooling function, this **transition time** can be expressed by:

$$t_{\text{trans}} \approx 3.2 \times 10^4 E_{\star}^{1/2} n_0^{1/2} \approx 10^4 \text{ [yrs]}$$

Finally, we see from the cooling runs that both simulations are able to describe correctly the snowplough phase since they both reproduce the **extension of the snowplough radius** as defined by *McKee & Ostriker-1977* (given by the green lines).



Simulation Setup

- 256³ SPH particles: $m_g = 0.01 M_{\text{sun}}$
 - $L_{\text{box}} \approx 85 \text{ pc}$ - $n_0 \approx 10 \text{ cm}^{-3}$
 - Energy input: $E_{\star} = 10^{51} \text{ erg}$
- +
Piecewise cooling curve

Outcome

As long as the resolution is high enough to **numerically converge** to the adiabatic phase before cooling processes become important, **concordance of feedback methods** is preserved.