FORECASTING THE CHEMICAL INFORMATION CONTENT OF STELLAR SPECTRA

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GOLDEN AGE OF STELLAR SPECTROSCOPY

Observations

- **Current Facilities**
  - Keck, VLT, Magellan
- **>10^5 stars outside the MW**
- **Next Generation Facilities**
  - Space-based: JWST
  - Ground-based: PFS, E-ELT, MSE

Techniques

- **Improved Spectral Models**
  - 3D, non-LTE
  - More complete linelists
- **Full Spectrum Fitting**
  - The Cannon (Ness+ 2015)
  - The Payne (Ting+ 2018)
  - StarNet (Fabbro+ 2017)
- **10+ Elements from R~2000 spectra in MW**

BUT WAIT…

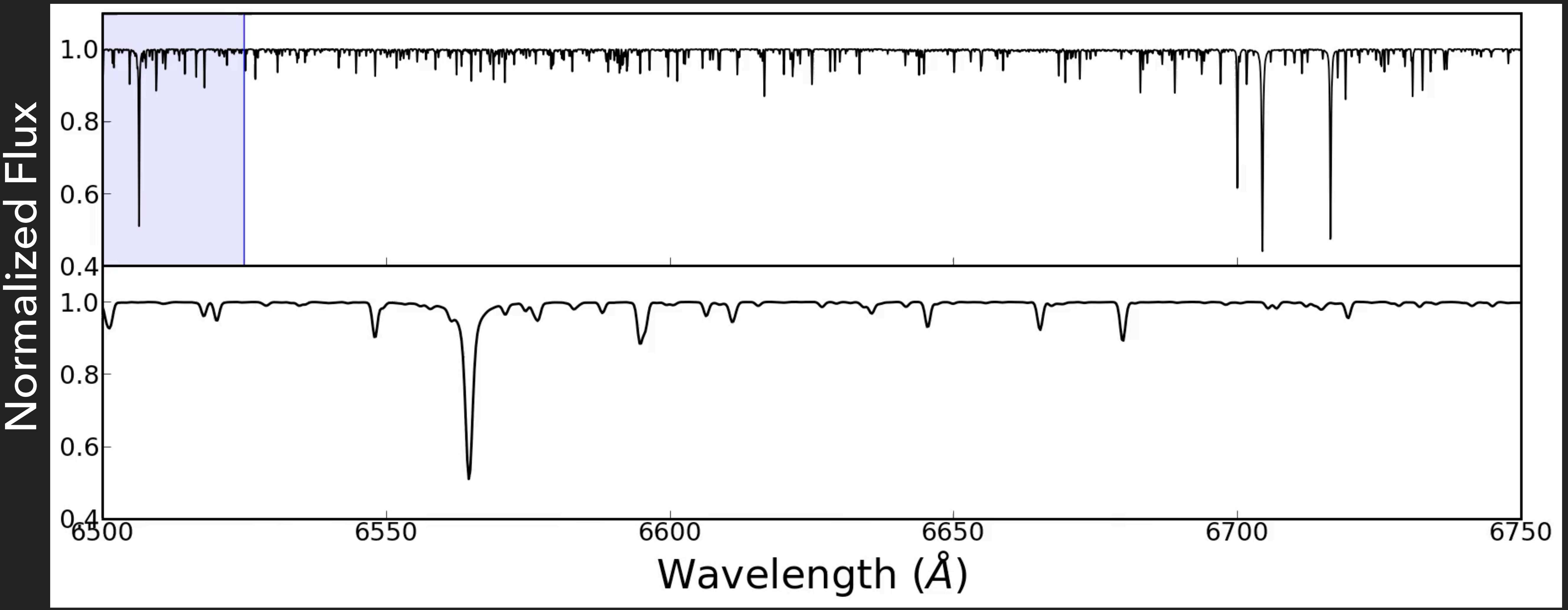
What abundances do we need?
To what precision?

Where is the information located?

What observations best provide
that information?

SYNTHETIC DEIMOS RGB SPECTRUM

$$6500\text{Å} < \lambda < 9000\text{Å}$$
$$R = 6500$$



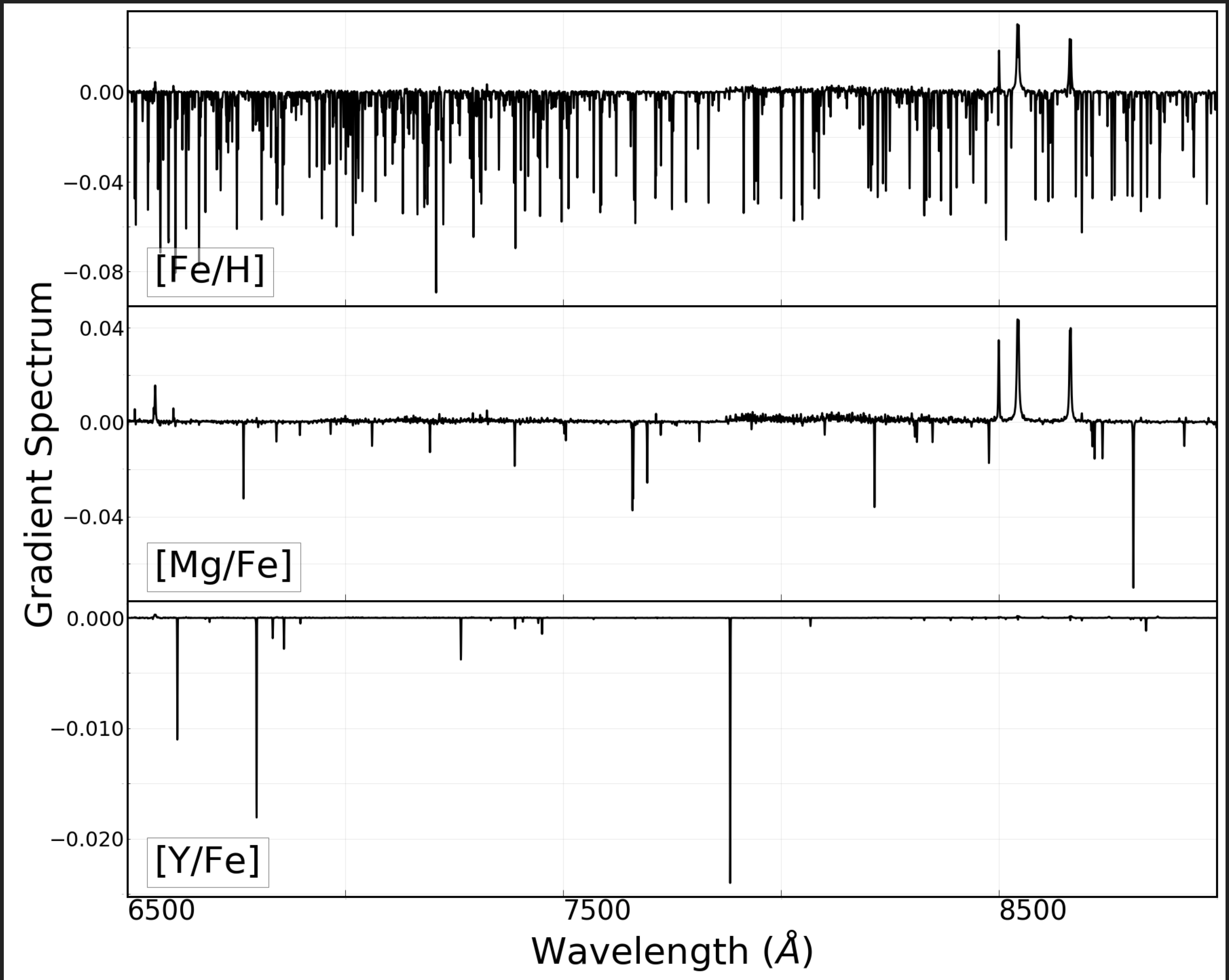
$$T_{\text{eff}} = 4750 \text{ K}$$
$$\log(g) = 1.8$$
$$v_{\text{turb}} = 1.9 \text{ km/s}$$
$$[\text{Fe/H}] = -1.5$$

CHEMICAL INFORMATION IN SPECTRAL GRADIENTS

Strong Gradients →

Moderate Gradients →

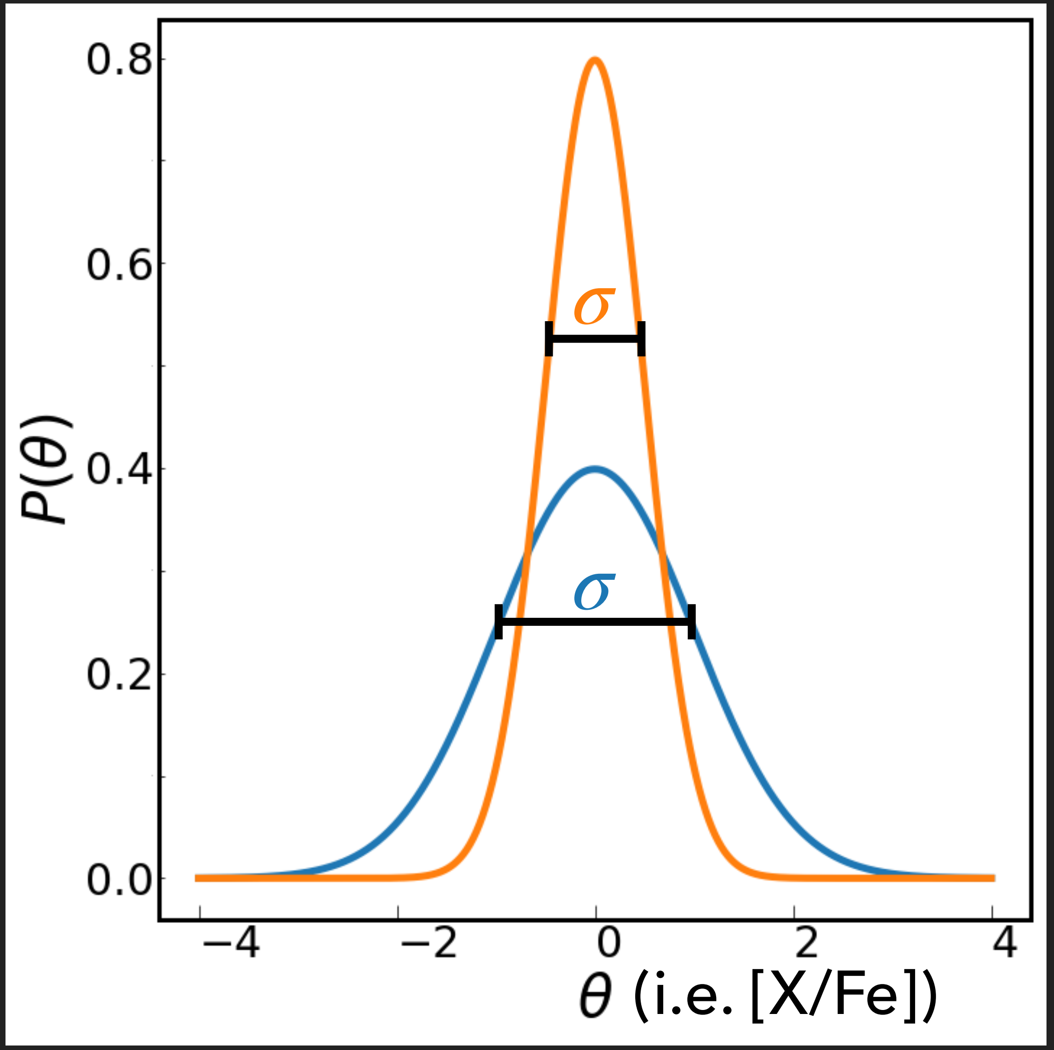
Weak Gradients →



QUANTIFYING CHEMICAL INFORMATION CONTENT

MCMC Sampling of the Posterior

- Abundance fitting is many-dimensional
- Computationally expensive / intractable

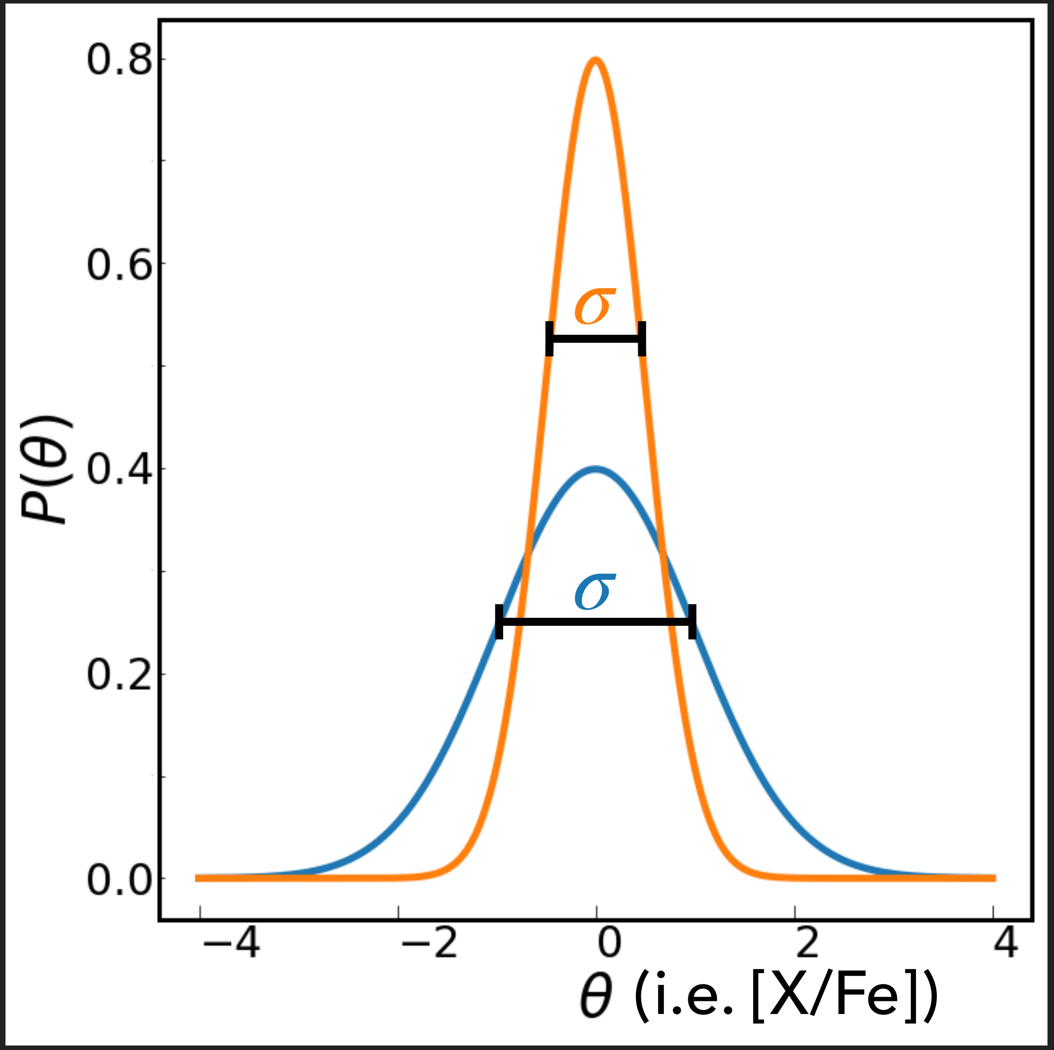


CRAMÉR–RAO LOWER BOUNDS

Fisher Information Matrix

- Negative curvature of the log-Posterior

$$F_{ij} = \left\langle \frac{\partial^2 [-\ln P(\theta)]}{\partial \theta_i \partial \theta_j} \right\rangle$$



CRAMÉR–RAO LOWER BOUNDS

Fisher Information Matrix

- Negative curvature of the log-Posterior
- Can be written in terms spectral gradients (Ting+ 2017)

$$F_{ij} = \underbrace{\left[\frac{\partial f(\lambda,\theta)}{\partial \theta_i}\right]^T}_{\text{Spectral Gradient wrt }\theta} \underbrace{\Sigma^{-1}}_{\text{S/N}} \underbrace{\left[\frac{\partial f(\lambda,\theta)}{\partial \theta_j}\right]}_{\text{Spectral Gradient wrt }\theta} \qquad \left(f(\lambda,\theta) = \text{Spectral Model}\right)$$

S/N

Spectral Gradient wrt θ

CRAMÉR–RAO LOWER BOUNDS

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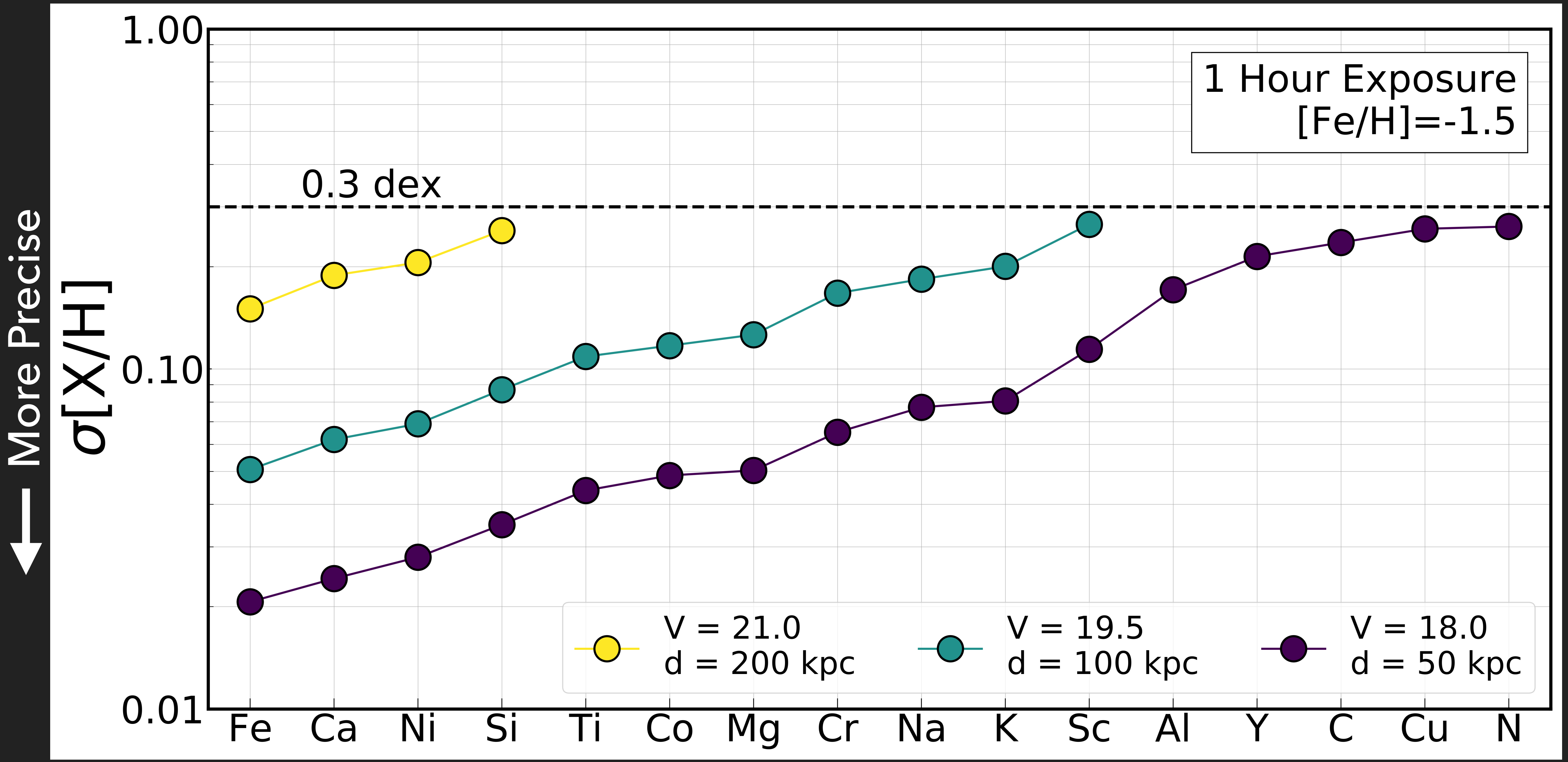
Cramér-Rao Inequality

$$\boxed{\sigma_i \leq \sqrt{(F^{-1})_{ii}}}$$

CRLB: Highest precision achievable from a set of observations

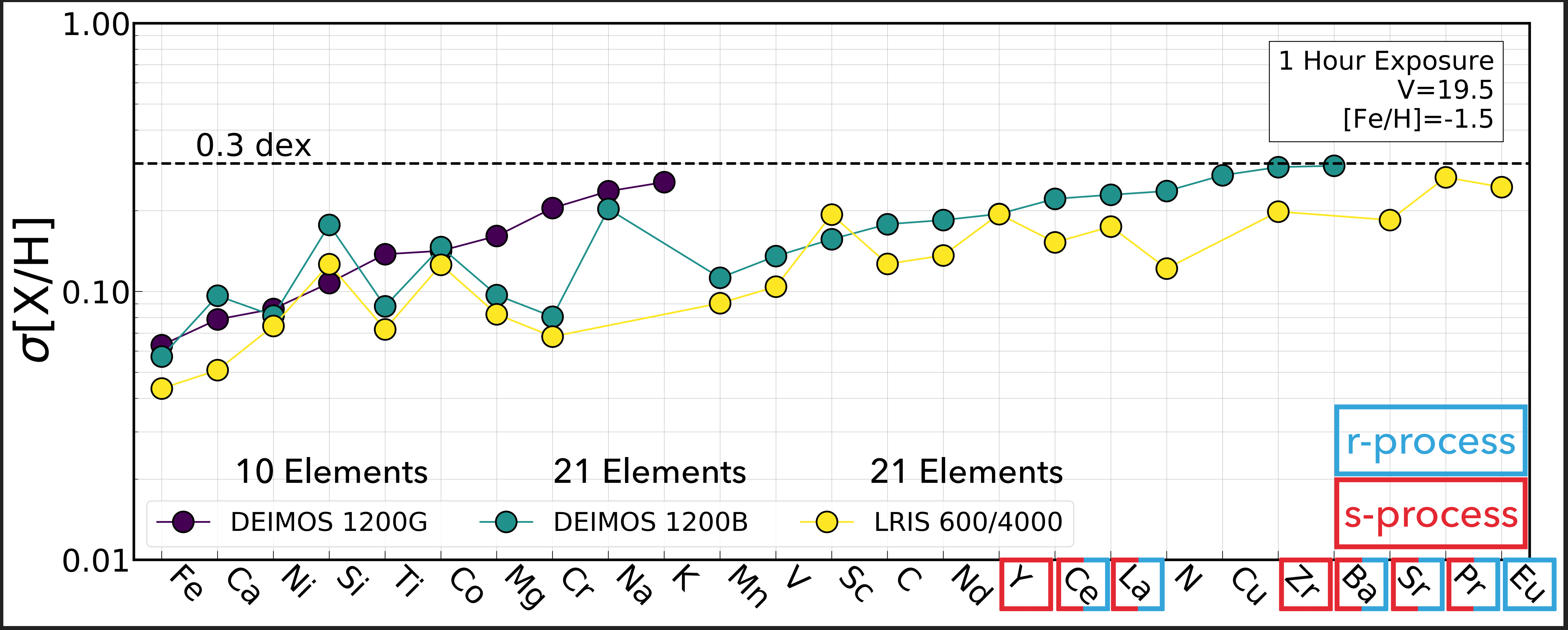
DEIMOS 1200G Theoretical Precision

$6500\text{Å} < \lambda < 9000\text{Å}$
$R = 6500$

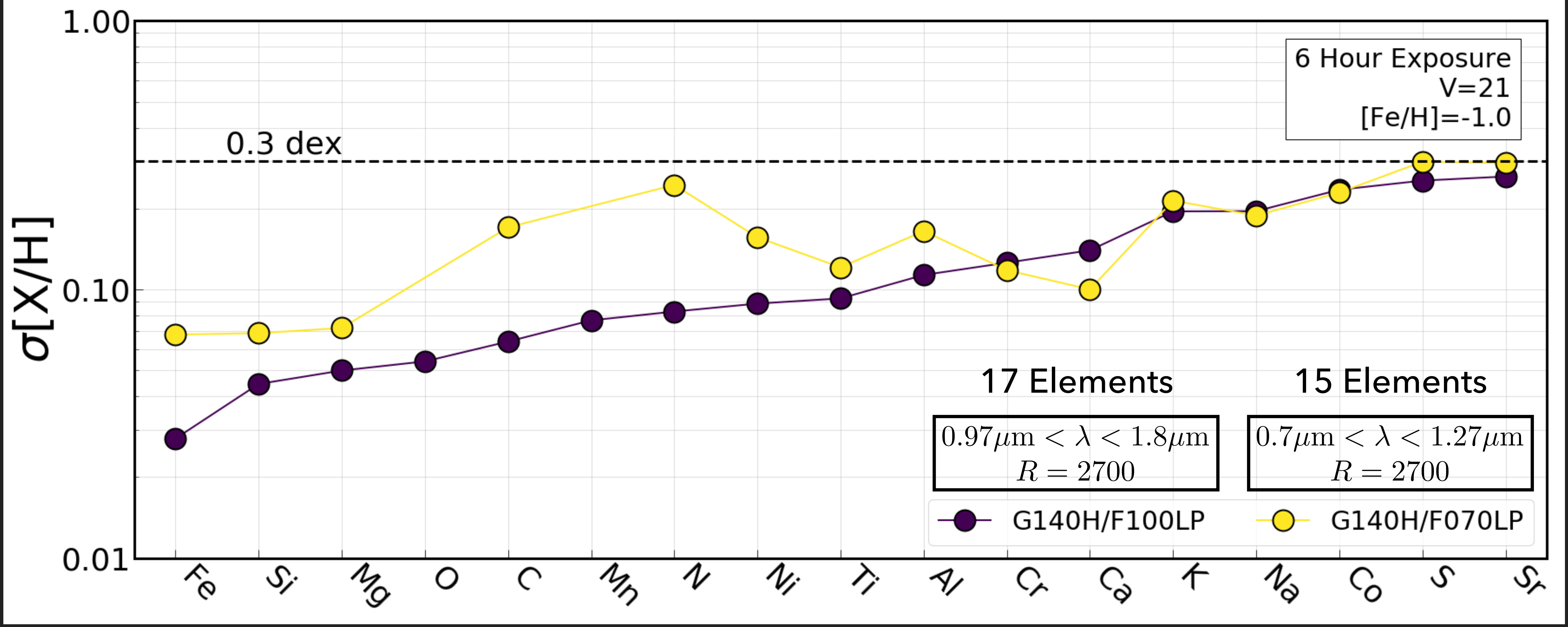


BLUE SPECTRA ARE INFORMATION RICH

DEIMOS 1200G	DEIMOS 1200B	LRIS 600/4000
$6500Å < \lambda < 9000Å$ $R = 6500$	$4000Å < \lambda < 6400Å$ $R = 4000$	$3900Å < \lambda < 5500Å$ $R = 1800$



JWST SPECTRA: FAINT STARS, CROWDED FIELDS



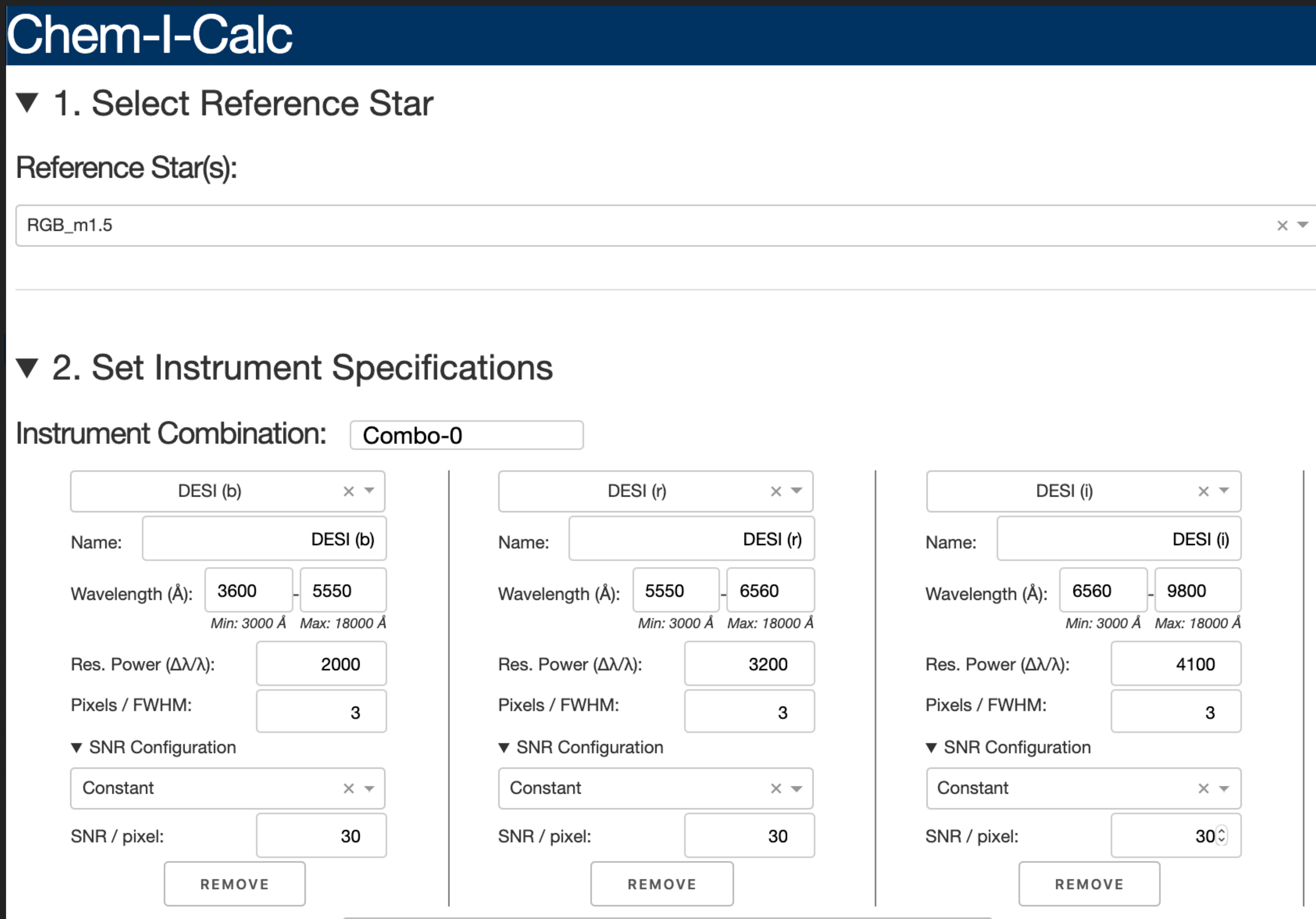
ARE THE CRLBS REALISTIC?

In Practice:

- Information lost
  - Detector gaps, Sky lines, Masked non-LTE lines
- Imperfect Calibrations
  - Continuum normalization, Sky subtraction, Wavelength solution
- Imperfect Models
  - Non-LTE lines, Incomplete linelists
- Model Interpolation
  - Many-dimensional fitting

These are all being actively improved upon!

Explore On Your Own!



CONCLUSION

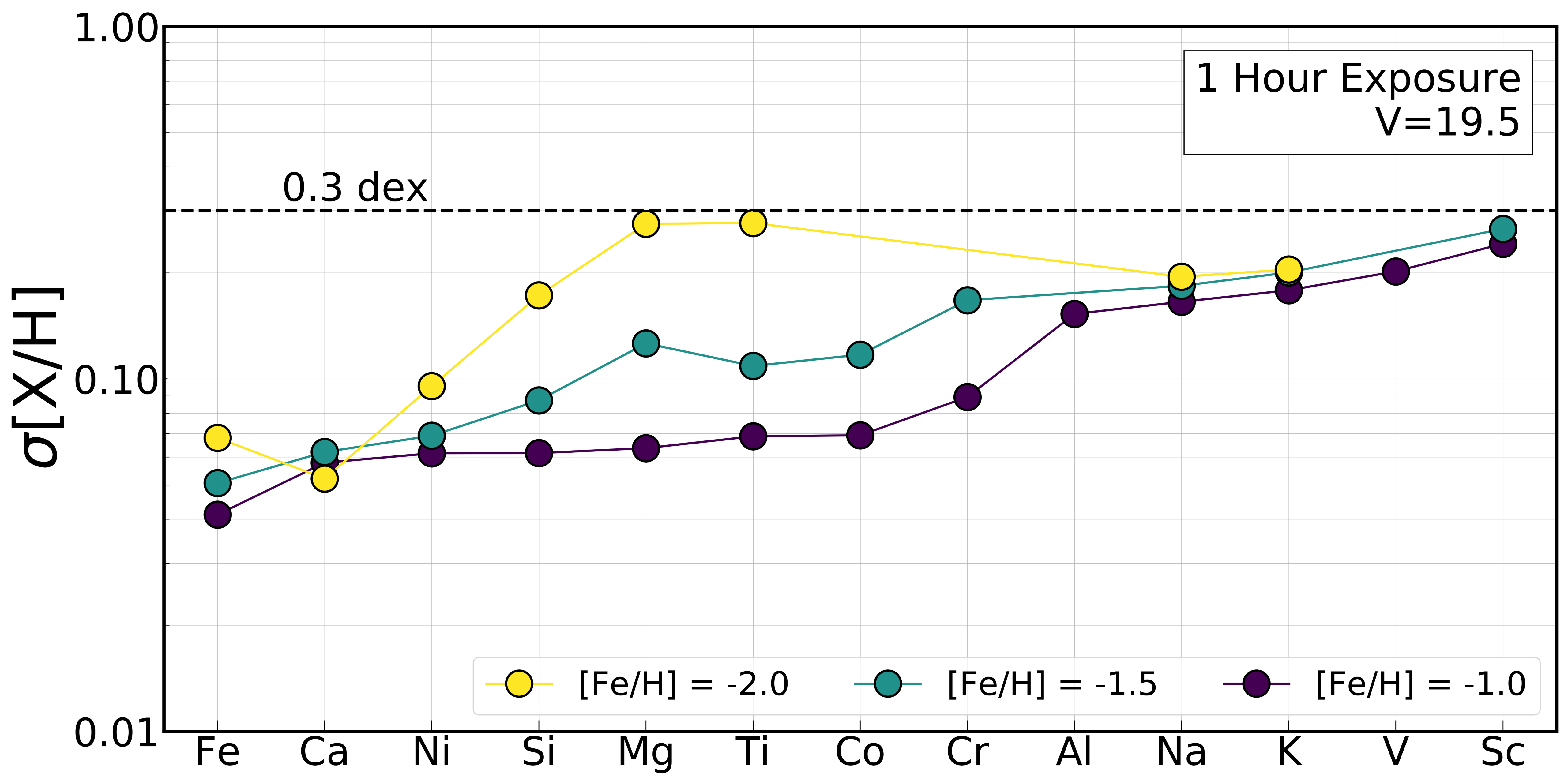
Take-Aways:

- CRLBs provide a quantitive measure of the information content of spectra
- Benchmark for achievable precision
- Useful tool for designing observations / instruments
- CRLBs are applicable to all spectral analysis (e.g. galaxies, HII regions)

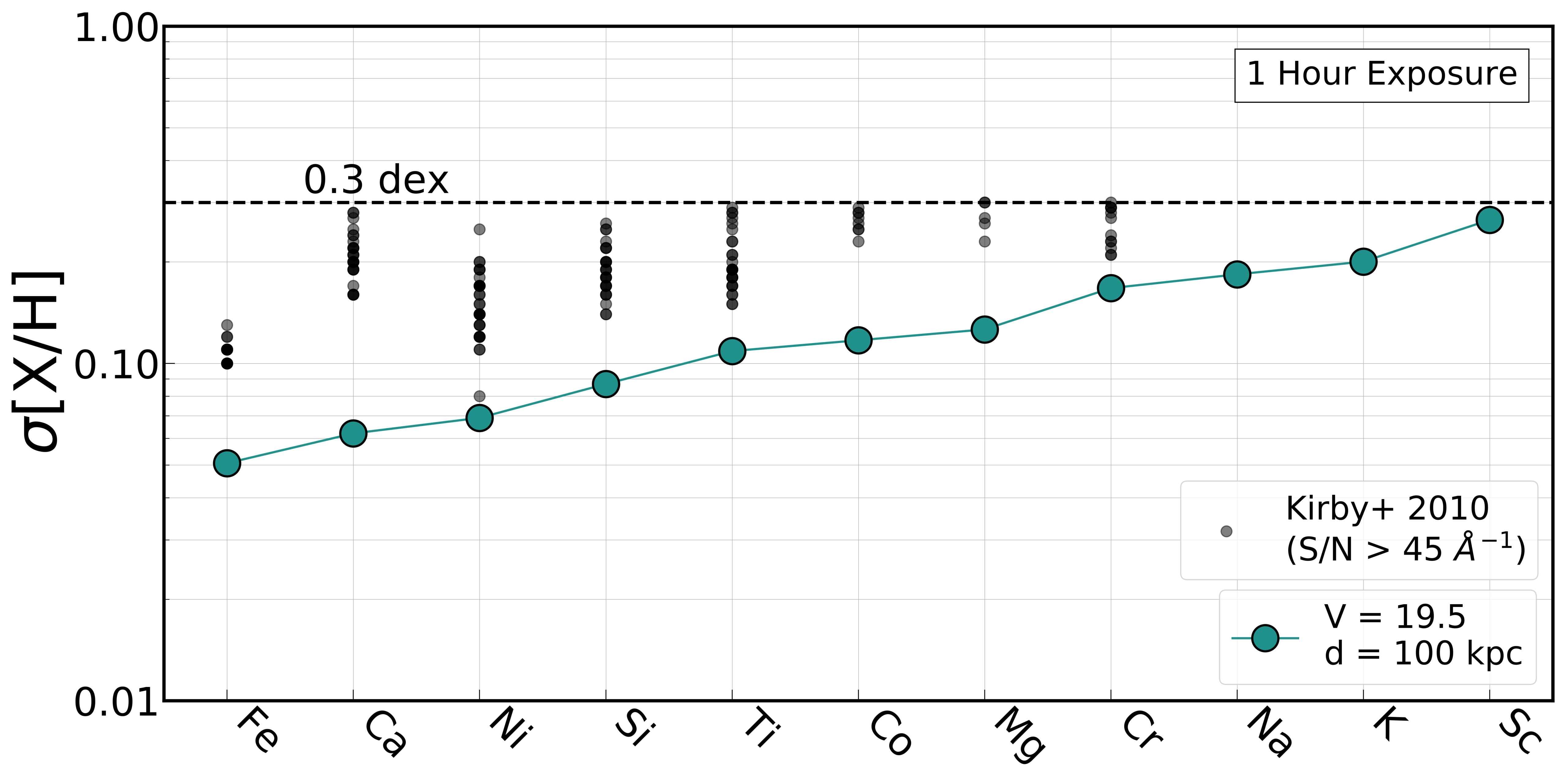
Explore on your own!

- Github: https://github.com/NathanSandford/Chem-I-Calc
- PyPi: `pip install Chem-I-Calc`
- Web Applet: Coming soon…

DEIMOS 1200G CRLB Metallicity Dependence



DEIMOS 1200G CRLBs v. Observations



BAYESIAN CRLBs

$$F_{ij} = \left[\frac{\partial f(\lambda, \theta)}{\partial \theta_i}\right]^T \Sigma^{-1} \left[\frac{\partial f(\lambda, \theta)}{\partial \theta_j}\right] + \boxed{F_{ij,prior}}$$

For Gaussian Priors:

$$F_{ii,prior} = \left(\frac{1}{\sigma_{i,\mathrm{prior}}}\right)^2$$

$$\boxed{\sigma_i \leq \sqrt{(F + \sigma_{prior}^{-2})_{ii}^{-1}}}$$