Phase Coherence of WMAP and Planck

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Outline

Complex Phases

Peter Coles, Chiang Etal

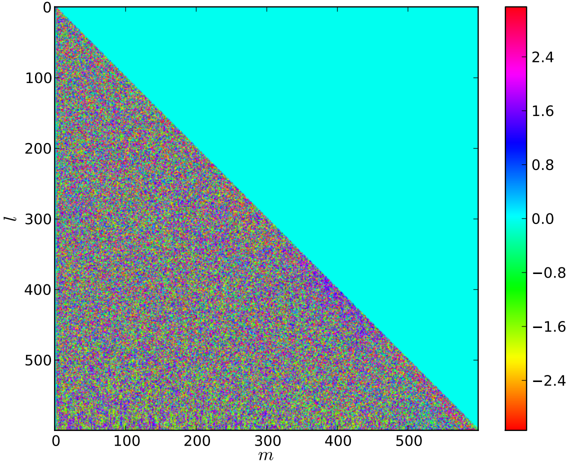
- Complex phases are defined by the CMB multipoles

$$a_{\ell m} = |a_{\ell m}| \cdot \exp(i\phi_{\ell m})$$

- They correspond to rotations around the z-axis
- A subgroup of the full SO(3)
- Phases are random for a Gaussian distribution
- Tests for non-Gaussianity

WMAP7 Complex Phase Diagram

Kovács, Szapudi & Frei (2013)



Generalized Phases

Kovács, Carron & Szapudi (2013)

- GP's use unit vectors in the $2\ell + 1$ dimensional representation spaces of SO(3)
-

$$\varepsilon_\ell = (a_{\ell 0}/\sqrt{2}, Re[a_{\ell 1}], ...Re[a_{\ell\ell}], Im[a_{\ell 1}], ...Im[a_{\ell\ell}])$$

- The amplitude of this vector is essentially the pseudo power spectrum
- The direction is the GP

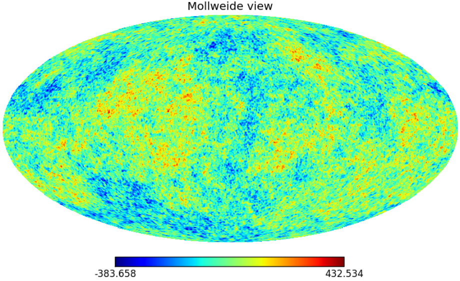
$$\hat{\varepsilon}_\ell = \frac{\varepsilon_\ell}{\sqrt{\sum_k \varepsilon_{\ell,k}^2}}.$$

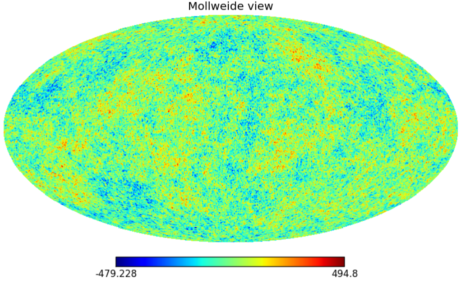
Generalized Phases

- GP's are random for Gaussian distributions
- For a Gaussian distribution the cosmological information is contained in the (pseudo)-power spectrum
- GP's are complementary to the power spectrum
- GP's + pseudo power spectrum determine a map, phases+pseudo-C_ℓ's do not
- Most non-Gaussianity would result in a degree of phase correlation
- Applications: constrain non-Gaussianity and compare coherence of two measurements

Randomizing phases

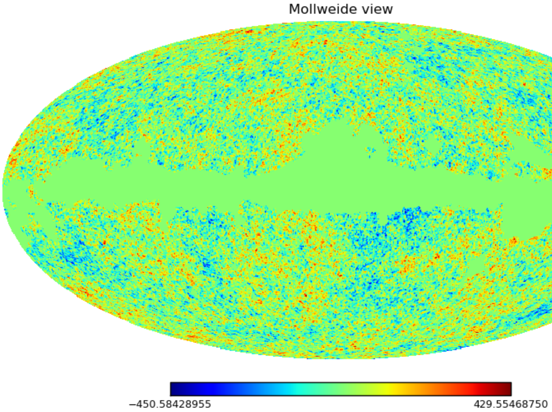
Left:simulation Right: Smica with Random GP's





Planck Smica

with mask degraded to $N_{side} = 512$



Data sets
WMAP+Planck

- Our goal is to quantify the coherance of WMAP and Planck due to observing the same CMB
- Gradual decoherence is expected due to noise
- WMAP QVW maps foreground reduced or not
- $N_{side} = 512$ HEALPix maps Temperature Analysis Masks
- Planck Smica (and NILC) downgraded to same resolution and same mask has been used
- Analysis has been done with HEALPix based SpICE as well as specific python programs

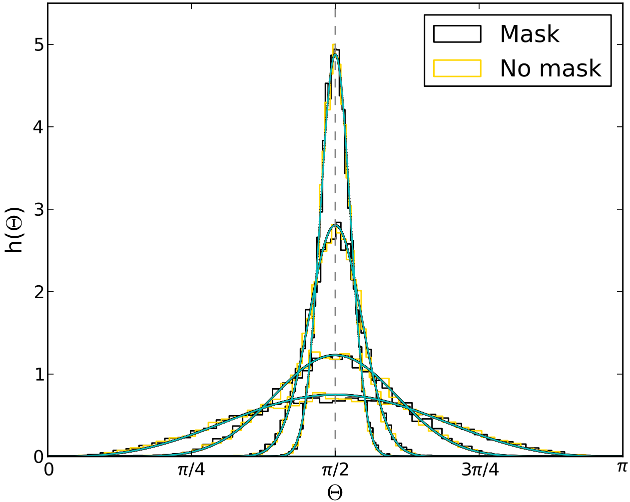
Random Phase Statistics
Cai etal (2013)

- Angles between GP's of two uncorrelated maps are

$$h_{n=2\ell+1}(\Theta) = \frac{1}{\sqrt{\pi}} \frac{\Gamma(\frac{n}{2})}{\Gamma(\frac{n-1}{2})} \cdot \sin^{n-2}\Theta.$$

- The above formula quickly tends to a unit Gaussian around 90°
- This allows us to form the null hypothesis that "two maps are uncorrelated"
- We aim to reject this at 5σ to show coherence
- Meaning of $\cos\theta$: correlation coefficient in $2\ell + 1$ dimensions.

Random Phase Distributions



Coherence between noisy maps

- When two maps are not random but differ by a random noise, we can calculate the phase distribution

$$h_N(\Theta) = \frac{\Gamma(n)}{\Gamma\left(\frac{n-1}{2}\right)} \sin^{n-2}\Theta \cdot \exp\left(-\frac{n}{2} SN^2 \sin^2\Theta\right) \mathrm{i}^{n-1}\mathrm{erfc}\left(-\sqrt{\frac{n}{2}} SN \cos\Theta\right)$$

- where

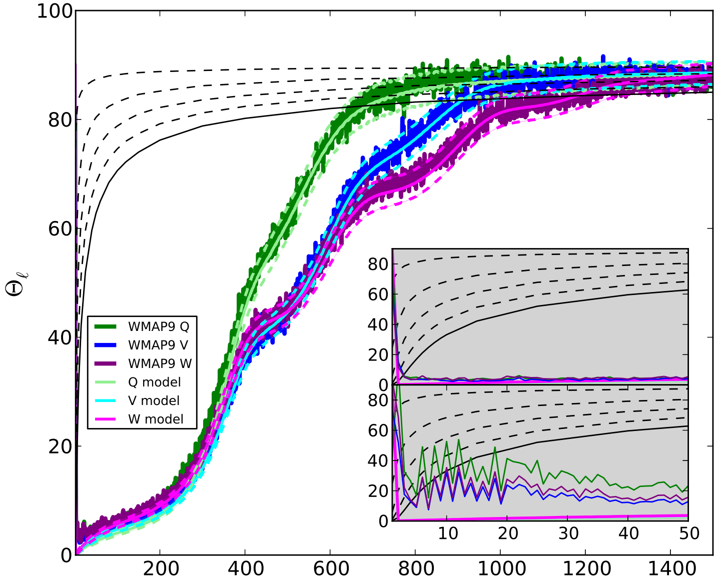
$$SN = \frac{\left|\epsilon_\ell^{\mathrm{CMB}}\right|}{\left|\epsilon_\ell^{\mathrm{noise}}\right|} = \sqrt{\frac{C_\ell^{\mathrm{CMB}}}{C_\ell^{\mathrm{noise}}}}$$

Results

- We estimated the angle between GP's for Planck Smica and WMAP

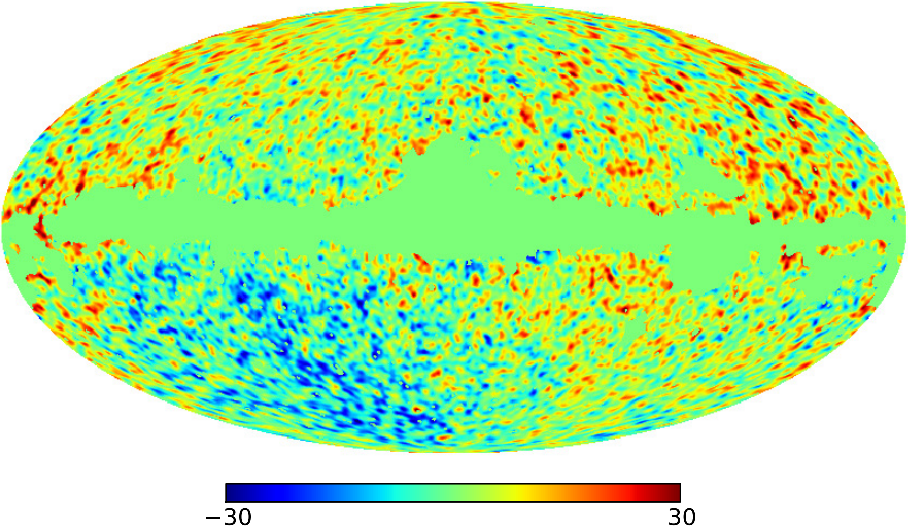
$$\cos\Theta_\ell = \sum_k \hat{\varepsilon}^{\rm Planck}_{\ell,k} \cdot \hat{\varepsilon}^{\rm WMAP}_{\ell,k}$$

- Additional meaning: Correlation Coefficient $C_\ell^{\rm WMAP,Planck} / \sqrt{C_\ell^{\rm WMAP} C_\ell^{\rm Planck}}$,
- 60° means 50% correlation


WMAP9 Q
WMAP9 V
WMAP9 W
Q model
V model
W model
Θ_ℓ
0
20
40
60
80
100
200
400
600
800
1000
1200
1400
10
20
30
40
50

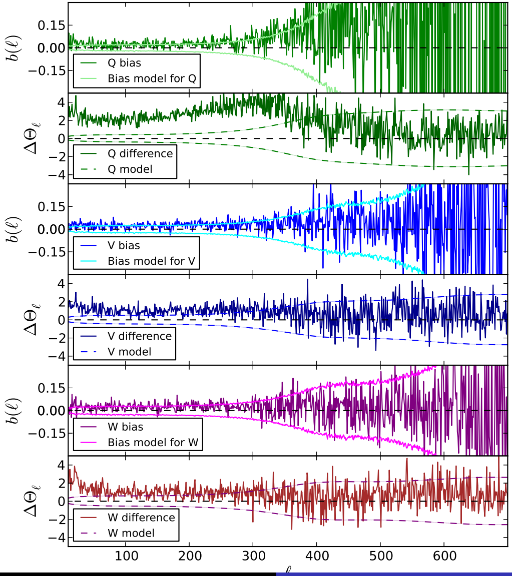
Interpretation

- Our measurement tightly tracks simulation and theory
- For the lowest ℓ's the no-correlation null hypothesis cannot be rejected at 5σ
- Foreground reduced maps increase coherence
- Dipole still dehoherent, but has no physical meaning
- Decoherence at $\ell \approx 700$, $\ell \approx 900$ and $\ell \approx 1100$ for Q, V and W maps, respectively.


−30
30

Detailed Interpretation

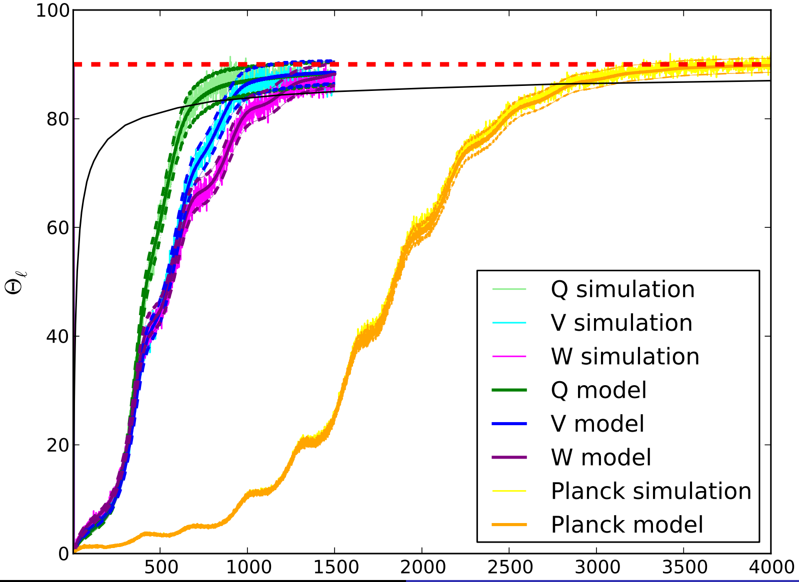
- Impact of mask at high ℓ: fully understood from theory and simulations
- For QVW, there is an excess decoherence for $\ell \lesssim 500, 400$ and 300, respectively.
- Our theory and simulation based on WMAP noise model predicts slightly more coherence
- At face value extra noise
- Checked pseudo power spectrum: about 2.6% higher for WMAP, significance 10's of σ
- Excess noise calculated for decoherence cannot fully explain the bias, except for Q, betwenn $250 < \ell < 500$
- Other possibilities, side-lobes?


0.15
0.00
−0.15
$b(\ell)$
Q bias
Bias model for Q
4
2
0
−2
−4
$\Delta\Theta_\ell$
Q difference
Q model
V bias
Bias model for V
V difference
V model
W bias
Bias model for W
W difference
W model
100
200
300
400
500
600
ℓ

Forecast

Planck vs CMB

- Using our theoretical tools we predicted the coherence of the Planck GP's with the true (noiseless) CMB
- Both simulations and theory forecast that decoherence starts around $\ell \lesssim 2900$
- It makes sense to for studies of non-Gaussianity to use lower ℓ's than this.


100
80
60
40
20
0
Θ_ℓ
500
1000
1500
2000
2500
3000
3500
4000
Q simulation
V simulation
W simulation
Q model
V model
W model
Planck simulation
Planck model

Summary

- ℓ-by-ℓ coherence of WMAP and Planck (Smica but NILC is same)
- New statistic, GP, a unit vector in the representation space
- Quantify coherence with the angle of two unit vectors
- Decoherence at 5σ in QVW at $\ell \lesssim 700, 900, 1100$
- 2.6% higher power in WMAP at very high significance
- might be related to excess decoherence up to $\ell \lesssim 500, 400, 300$
- excess noise from decoherence only explains Q $250 \lesssim \ell \lesssim 500$
- therefore quantitatively not a full explanation, need to dive into data
- understood even subtle effect of the mask at high ℓ
- forecast for Planck decoherence with the true CMB
- future: use GP's for constraining non-Gaussianity