University of Durham

EXAMINATION PAPER

May/June 2005 044161/01

Examination for the Degree of M.Sci.

LEVEL 4 PHYSICS: ADVANCED ASTROPHYSICS

SECTION A. GALAXY FORMATION
SECTION B. HIGH ENERGY ASTROPHYSICS
SECTION C. PHYSICAL PROCESSES IN THE ISM

Time allowed : 3 hours

Answer the compulsory question that heads each of sections A, B and C. These **three** questions have a total of 12 parts and carry 50% of the total marks for the paper. Answer **any three** of the six optional questions. The marks shown in brackets for the main parts of each question are given as a guide to the weighting that the markers expect to apply.

ANSWER EACH SECTION IN A SEPARATE ANSWER BOOK

APPROVED TYPES OF CALCULATOR MAY BE USED.

Information

$e = 1.60 \times 10^{-19}$ C	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
$k_{\mathrm{B}} = 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}$	$m_{\mathrm{e}} = 9.11 \times 10^{-31}$ kg
$G = 6.67 \times 10^{-11}\ \mathrm{N\,m^{2}\,kg^{-2}}$	$m_{\mathrm{p}} = 1.67 \times 10^{-27}$ kg
$h = 6.63 \times 10^{-34}$ J s	$\epsilon_{\mathrm{o}} = 8.85 \times 10^{-12}\ \mathrm{F\,m^{-1}}$
Bohr magneton $= 9.27 \times 10^{-24}\ \mathrm{J\,T^{-1}}$	$\mu_{\mathrm{o}} = 4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Nuclear magneton $= 5.05 \times 10^{-27}\ \mathrm{J\,T^{-1}}$	$R = 8.31 \times 10^{3}\ \mathrm{J\,K^{-1}\,kmol^{-1}}$
Avogadro's Constant $= 6.02 \times 10^{26}\ \mathrm{kmol^{-1}}$	$g = 9.81\ \mathrm{m\,s^{-2}}$
Stefan-Boltzmann Constant $= 5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$	

SECTION A. GALAXY FORMATION

Question 1 is compulsory. Questions 2 and 3 are optional

1. (a) Explain in words the physical signficance of (i) the Jean's Mass and (ii) the Meszaros effect. [5 marks]

   (b) In the expanding universe, the first law of thermodynamics takes the form:
   $$\frac{d\rho}{dt} + 3H\left(\rho + \frac{P}{c^2}\right) = 0$$
   where ρ and P are the density and pressure, and H is the Hubble parameter. Show that for a universe dominated by relativistic particles or radiation, $\rho \propto a^{-4}$. [5 marks]

   (c) Consider a universe dominated by the cosmological constant, Λ. The Friedmann equations may be written as:
   $$\left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} = \frac{\Lambda c^2}{3}$$
   and
   $$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho_\Lambda + \frac{3P}{c^2}\right),$$
   where a is the expansion factor, k the curvature of the universe, $\rho_\Lambda c^2$ the energy density associated with Λ and P the pressure. Writing the equation of state for Λ as $P = w\rho_\Lambda c^2$ and recalling that $\Lambda c^2 = 8\pi \mathrm{G}\rho_\Lambda$, show that in a Λ-dominated universe $w = -1$. [5 marks]

   (d) Estimate the Hubble radius at recombination in a flat, matter-dominated universe. How does the proper size of this region at the present day compare with today's Hubble radius? [5 marks]

   (e) What is the dominant contribution to the temperature anisotropy of the Cosmic Microwave Background on large scales (ignoring any effects due to our galaxy's motion)? Recalling that photons emitted from a potential well at the epoch of recombination experience both a gravitational redshift and a time dilation, derive an expression for the Sachs-Wolfe effect. [5 marks]

2. The equation that describes the linear growth of a density perturbation in a non-relativistic fluid of density ρ and pressure P in the matter era is
$$\ddot{\delta} + 2H\dot{\delta} = \frac{\nabla_r^2(\delta P)}{\overline{\rho}} + \nabla_r^2\phi,$$
where
$$\nabla_r^2\phi = 4\pi\mathrm{G}(\delta\rho_{\mathrm{tot}}).$$
Here, H is the Hubble parameter, $\overline{\rho}$ the mean density, $\delta \equiv \delta\rho/\overline{\rho}$ the density perturbation, ϕ the perturbation in the gravitational potential, (δP) the perturbation in the pressure and $(\delta\rho_{\mathrm{tot}})$ the perturbation in the *total* density.

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(a) Show that for cold matter (dust), the perturbation equation reduces to

$$\ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}H^2\Omega_\mathrm{m}\delta = 0 \qquad (1)$$

where Ω_m is the matter density parameter. [2 marks]

Derive the growing and decaying mode solutions of equation (1) for a universe with critical matter density ($\Omega_\mathrm{m} = 1$). [8 marks]

(b) Consider a universe dominated by a uniform dark energy field for which the Friedmann equation is

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} = \frac{\Lambda c^2}{3},$$

where k is the curvature and Λ the cosmological constant. Show that density fluctuations in a flat universe of this kind do not grow. [6 marks]

(c) Show that, whether primordial fluctuations are adiabatic or isothermal, the measured amplitude of temperature fluctuations in the microwave background radiation implies that collapsed structures such as galaxy clusters can only exist today if the dark matter is predominantly non-baryonic. [4 marks]

3. Consider a uniform spherical overdense region of mass M and radius r in an Einstein-de-Sitter universe. A parametric solution of the equations of motion of this region is:

$$r = A(1 - \cos\eta) \qquad t = B(\eta - \sin\eta),$$

where A and B are constants.

(a) By considering the behaviour of this solution at early times ($\eta << 1$, $r \propto t^{2/3}$, mean cosmic density $\bar{\rho} \propto M/r^3$), show that

$$A^3 = GMB^2,$$

where G is Newton's gravitational constant. [8 marks]

(b) By taking into account that the mass of the overdense region is conserved, show that when the region turns around, its density is $9\pi^2/16$ times the critical density. [6 marks] If the region reaches maximum expansion at $t = t_\mathrm{ta}$, show that it collapses to a singularity at $t = 2t_\mathrm{ta}$. [2 marks]

(c) Assume that the age of the Universe is 10^{10} years and that the Coma cluster (of mass $10^{15}M_\odot$) has only just collapsed. Calculate the turnaround radius of Coma. [4 marks]

[$M_\odot = 1.99 \times 10^{30}$ kg; 1 pc $= 3.09 \times 10^{16}$ m; 1 year $= 3.15 \times 10^7$ s]