a) HDM: particles that are relativistic when they decouple. Free-streaming wipes out fluctuations smaller than a critical wavelength.

CDM: particles that are non-relativistic when they decouple. Fluctuations survive on ~~most~~ all scales of interest. [BOOKWORK]

HDM is ruled out because this model cannot simultaneously reproduce the ages of galaxies and quasars and their clustering strength today. 5

b) For radiation: $p = \frac{1}{3}\rho c^2 \Rightarrow \frac{d\rho}{dt} + 3H\,\frac{4}{3}\rho = 0 \Rightarrow \frac{d\rho}{da}\dot{a} + 4\frac{\dot{a}}{a}\rho = 0$

$\Rightarrow \frac{d\rho}{\rho} = -4\frac{da}{a} \Rightarrow \rho \propto a^{-4}$ [BOOKWORK / UNSEEN]

For non-rel. matter, $p = 0 \Rightarrow \frac{d\rho}{dt} + 3H\rho = 0 \Rightarrow \rho \propto a^{-3}$. 5

c) $\nabla_r^2\,\delta p = \frac{1}{a^2}\nabla_x^2\,\delta p = \frac{c_s^2}{a^2}\nabla_x^2\,\delta\rho = -\frac{k^2 c_s^2}{a^2}\delta\rho = -\frac{k^2c^2}{a^2}\bar{\rho}\,\delta$

[BOOKWORK]

$\therefore\ \ddot{\delta} + 2H\dot{\delta} = -\frac{k^2c^2}{a^2}\frac{\bar{\rho}\,\delta}{\bar{\rho}} + 4\pi G\bar{\rho}\delta = \left(4\pi G\bar{\rho} - \frac{k^2c^2}{a^2}\right)\delta$ 5

Fluctuations can only grow if RHS > 0, i.e. for physical wavelength $\lambda = \frac{2\pi a}{k}$ greater than the Jeans wavelength $\lambda_J = c_s\sqrt{\frac{\pi}{G\bar{\rho}}}$

Physical interpretation: λ_J corresponds to balance between gravity & pressure forces

d) The radius of the pert. at horizon entry is the Hubble radius,

[BOOKWORK / UNSEEN]

$R_H = \frac{c}{H}$. $H \propto \frac{1}{t} \Rightarrow \frac{H}{H_0} = \frac{t_0}{t} \Rightarrow R_H(t_{eq}) = \frac{c}{H_0}\frac{t_{eq}}{t_0}$ 5

Now, $a \propto t^{2/3} \propto (1+z)^{-1} \Rightarrow \frac{t_{eq}}{t_0} = \frac{1}{(1+z_{eq})^{3/2}} \Rightarrow R_H(t_{eq}) = \frac{c}{H_0}\frac{1}{(1+z_{eq})^{3/2}}$

$\therefore\ R_H(t_{eq}) = \frac{3\times10^5\ \mathrm{km/s}}{70\ \frac{\mathrm{km}}{\mathrm{s\,Mpc}}}\ \frac{1}{(1.3\times10^4)^{3/2}} = 0.003\ \mathrm{Mpc} = 3\ \mathrm{kpc}$

Amplitude $\delta \propto a$ $\therefore$ $\delta(t_{rec}) = \delta(t_{eq})\frac{a_{rec}}{a_{eq}} = 10^{-6}\times\frac{1.3\times10^4}{10^3} = 1.3\times10^{-5}$

e) $\dfrac{\rho_h}{\bar\rho_c} = 178.$ Now $\bar\rho_0 = \bar\rho_c\,(1+z_c)^{-3}$ but $\rho_h(z_c) = \rho_h(z_0)$

So $\dfrac{\rho_h(z_0)}{\bar\rho_0} = \dfrac{\rho_h}{\bar\rho_c}(1+z_c)^3 = 178\,(1+z_c)^3$ [UNSEEN]

Now: $\sigma^2 \simeq \dfrac{GM}{r_h}$ and $\rho_h = \dfrac{M}{\frac{4}{3}\pi r_h^3}$ $\Rightarrow \frac{4}{3}\pi r_h^2 \rho_h = \dfrac{M}{r_h}$

So $\sigma^2 = \frac{4}{3}\pi G r_h^2 \rho_h = \frac{4}{3}\pi G r_h^2 \bar\rho_0\, 178\,(1+z_c)^3$ 5

But $\Omega H^2 = \frac{8\pi}{3} G\bar\rho_0$ $\Rightarrow$ $\sigma^2 = \frac{4}{3}\pi G r_h^2 \dfrac{\Omega H^2}{\frac{8\pi}{3}G}\, 178\,(1+z_c)^3$

$\Rightarrow \sigma^2 = \frac{1}{2}\Omega H^2 r_h^2\, 178\,(1+z_c)^3 = 90\, r_h^2\, \Omega H^2\,(1+z_c)^3$