Soln. Solution to QUESTION 2 MAY/JUNE 2006

(a) Dust universe $\Rightarrow \delta p = 0$

$$\frac{\partial}{\partial t} = \dot a \frac{\partial}{\partial a} \quad ; \quad \frac{\partial^2}{\partial t^2} = \ddot a \frac{\partial}{\partial a} + \dot a \dot a \frac{\partial^2}{\partial a^2}$$ [UNSEEN] 2

$$\therefore \quad \dot a^2 \delta'' + \ddot a \delta' + 2 \frac{\dot a}{a} \dot a \delta' = 4\pi G \delta\rho$$

$$\Rightarrow \quad \delta'' + \frac{\ddot a}{\dot a^2}\delta' + \frac{2}{a}\delta' = \frac{4\pi G \bar\rho \delta}{\dot a^2} \ ; \quad \text{but } \Omega H^2 = \frac{8\pi}{3} G\bar\rho \text{ and } \bar q = -\frac{a\ddot a}{\dot a^2}$$

$$\Rightarrow \quad \delta'' - \frac{q}{a}\delta' + \frac{2}{a}\delta' = \frac{4\pi G}{\dot a^2}\frac{3\Omega}{8\pi G}\left(\frac{\dot a}{a}\right)^2 \delta = \frac{3}{2}\frac{\Omega}{a^2}\delta$$ 6

$$\Rightarrow \quad \delta'' + \frac{2-q}{a}\delta' - \frac{3}{2}\frac{\Omega}{a^2}\delta = 0$$

(b) Try $\delta = A a^n \Rightarrow \delta' = A n a^{n-1}$ and $\delta'' = A n(n-1) a^{n-2}$ 2

[UNSEEN but similar to set problem]

$$\therefore \quad A n (n-1) a^{n-2} + \frac{2-q}{a} A n a^{n-1} - \frac{3}{2} A \Omega \frac{a^n}{a^2} = 0$$

Flat $\Rightarrow \Omega = 1, \ q = 1/2 \quad \Rightarrow n(n-1) a^{n-2} + (2-q) n a^{n-2} - \frac{3}{2}\Omega a^{n-2} = 0$

$$\Rightarrow \quad n(n-1) + (2-q)n - \frac{3}{2}\Omega = n^2 - n + (2-q)n - \frac{3}{2}\Omega = n^2 + (1-q)n - \frac{3}{2}\Omega = 0$$

$$\Rightarrow \quad n = \frac{-(1-q) \pm \sqrt{(1-q)^2 - 4(-\frac{3}{2}\Omega)}}{2} = \frac{-\frac{1}{2} \pm \sqrt{\frac{1}{4} + 6}}{2} = \frac{-\frac{1}{2} \pm \sqrt{\frac{25}{4}}}{2} = \frac{-\frac{1}{2} \pm \frac{5}{2}}{2}$$ 2

$$\Rightarrow \quad n = \begin{cases} 1 \\ -\frac{3}{2} \end{cases} \quad \therefore \quad \delta = A a + B a^{-3/2}$$

[UNSEEN]

(c) $\Omega = 0$ (no matter) and $q = -1$.

(1) $\Rightarrow \quad \delta'' + \frac{3}{a}\delta' = 0 \quad \Rightarrow \quad \frac{d\delta'}{\delta'} = -3\frac{da}{a} \quad \Rightarrow \quad \delta' = A a^{-3}$ 2

$\therefore \quad d\delta = A a^{-3} da \quad \Rightarrow \quad \delta = -\frac{A}{2} a^{-2} + B$ 2

[UNSEEN]

(d) $\frac{\delta\rho}{\rho} \sim \frac{\delta T}{T} \sim 10^{-5}$ at $z_{rec} \sim 10^3$. In a flat u., $\delta \propto a$. 4

$\therefore$ Today $\left(\frac{\delta\rho}{\rho}\right)_{bar} \sim 10^{-5} \times 10^3 \sim 10^{-2} < 1$. Collapse requires $\frac{\delta\rho}{\rho} \geq 1$

For $\Omega_{bary} < 1$, growth is even less $\Rightarrow$ DM must be non-baryonic.