Soln

Sollution to Question 3 May/JUNE 2006

(a) The power spectrum is defined as $P(\underline{k}) = |\delta_k(\underline{k})|^2$, where δ_k are the coefficients of the Fourier representation of the density fluctuation field, δ. A Gaussian random field is one in which the Fourier waves have random phases. [BOOKWORK] 4

(b) Choose a coordinate system with $\hat{z}$ in the $\underline{r}$ direction. Then,

$$\xi(r) = \frac{1}{(2\pi)^3} \int P(k)\, e^{-ikr\cos\theta}\, k^2 \sin\theta\, d\theta\, d\varphi\, dk$$

$$\therefore \xi(r) = \frac{1}{(2\pi)^2} \int P(k) k^2 dk\, (-1) \int_1^{-1} e^{-ikr\cos\theta}\, d(\cos\theta) = \frac{1}{(2\pi)^2} \int P(k) k^2 dk\, (-1) \left. \frac{e^{-ikr}}{-ikr} \right|_1^{-1}$$

$$= \frac{1}{(2\pi)^2} \int P(k) k^2 dk \cdot \frac{e^{ikr} - e^{-ikr}}{ikr} = \frac{1}{(2\pi)^2} \int P(k) k^2 dk\, \frac{2\cancel{i}\sin kr}{\cancel{i}kr}$$

$$\therefore A = \frac{1}{2\pi^2} \quad \text{and} \quad B = \frac{\sin kr}{kr}$$

[UNSEEN, BUT CLOSE TO A SET PROBLEM] 4

(c) $\sigma \propto k^{\frac{3+n}{2}}$; but $k \propto \lambda^{-1}$ and $M \propto \lambda^3$ so

[UNSEEN]

$$\sigma \propto \lambda^{-\frac{3+n}{2}} \propto \lambda^{-3\alpha} \Rightarrow -3\alpha = -\frac{3+n}{2} \Rightarrow \alpha = \frac{1}{2} + \frac{n}{6}$$

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Mass invariant fluctuations $\Rightarrow \alpha = 0 \Rightarrow \frac{n}{6} = -\frac{1}{2} \Rightarrow \underline{n = -3}$

(d) From (b), $\xi(r) = \frac{1}{2\pi^2} \int_0^\infty P(k) k^2 \frac{\sin kr}{kr} dk \propto \frac{1}{2\pi^2} \int_0^\infty k^{2+n} \frac{\sin kr}{kr} dk$

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$$\therefore \xi(r) \propto \frac{1}{r^{2+n}\, r} \int_0^\infty (kr)^{2+n} \frac{\sin(kr)}{(kr)}\, d(kr) \propto r^{-(3+n)}$$

BOOKWORK/UNSEEN

(e) $\frac{\sigma_{rec}}{\sigma_H} = \frac{a_{rec}}{a_H} \Rightarrow \sigma_{rec} = \sigma_H \frac{a_{rec}}{a_H} \propto a_H^{-1}$ 4

Now, $M_H \propto \bar{\rho}_H (ct_H)^3 \propto a_H^{-3} (a_H^{3/2})^3 \propto a_H^{3/2} \Rightarrow a_H \propto M_H^{2/3}$

$\therefore \sigma_{rec} \propto M_H^{-2/3}$. From (c) $\alpha = \frac{1}{2} + \frac{n}{6} = +\frac{2}{3} \Rightarrow n = 6\left(+\frac{2}{3} - \frac{1}{2}\right) = \underline{+1}$