University of Durham

EXAMINATION PAPER

May/June 2006 044161/01

Examination for the Degree of M.Sci.

LEVEL 4 PHYSICS: ADVANCED ASTROPHYSICS

SECTION A. GALAXY FORMATION
SECTION B. HIGH ENERGY ASTROPHYSICS
SECTION C. PHYSICAL PROCESSES IN THE ISM

Time allowed : 3 hours

Answer the compulsory question that heads each of sections A, B and C. These **three** questions have a total of 12 parts and carry 50% of the total marks for the paper. Answer **any two** of the four optional questions. The marks shown in brackets for the main parts of each question are given as a guide to the weighting that the markers expect to apply.

ANSWER EACH SECTION IN A SEPARATE ANSWER BOOK

APPROVED TYPES OF CALCULATOR MAY BE USED.

Information

$e = 1.60 \times 10^{-19}$ C	$c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
$k_B = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$	$m_e = 9.11 \times 10^{-31}$ kg
$G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$	$m_p = 1.67 \times 10^{-27}$ kg
$h = 6.63 \times 10^{-34}$ J s	$\epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Bohr magneton $= 9.27 \times 10^{-24}\,\mathrm{J\,T^{-1}}$	$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$
Nuclear magneton $= 5.05 \times 10^{-27}\,\mathrm{J\,T^{-1}}$	$R = 8.31 \times 10^{3}\,\mathrm{J\,K^{-1}\,kmol^{-1}}$
Avogadro's Constant $= 6.02 \times 10^{26}\,\mathrm{kmol^{-1}}$	$g = 9.81\,\mathrm{m\,s^{-2}}$
Stefan-Boltzmann Constant $= 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$	

SECTION A. GALAXY FORMATION

Question 1 is compulsory. Questions 2 and 3 are optional

1. (a) Explain the difference between hot and cold dark matter. Why is hot dark matter ruled out as the dominant form of dark matter? [5 marks]

(b) In the expanding universe, the first law of thermodynamics takes the form

$$\frac{d\rho}{dt} + 3H\left(\rho + \frac{P}{c^2}\right) = 0$$

where ρ and P are the density and pressure, and H is the Hubble parameter. Show that for a universe dominated by relativistic particles or radiation, $\rho \propto a^{-4}$ and for a universe dominated by non-relativistic particles, $\rho \propto a^{-3}$. [5 marks]

(c) The equation that describes the linear growth of a density perturbation in a gravitationally dominant, non-relativistic fluid of density ρ and pressure P in the matter era is

$$\ddot{\delta} + 2H\dot{\delta} = \frac{\nabla^2_{\underline{r}}(\delta P)}{\overline{\rho}} + 4\pi \mathrm{G}\,(\delta\rho)$$

Here, (δP) and $(\delta\rho)$ denote the perturbations in the pressure and density respectively, $\delta \equiv \delta\rho/\overline{\rho}$, $\overline{\rho}$ is the mean cosmic density and H the Hubble parameter. Consider a plane-wave adiabatic perturbation in a perfect baryon fluid of the form

$$\delta P = c_s^2(\delta\rho) \propto e^{i\underline{k}\cdot\underline{x}}$$

where c_s is the sound speed. Show that the Jeans length is given by:

$$\lambda_J = c_s\left(\frac{\pi}{\mathrm{G}\overline{\rho}}\right)^{1/2}$$

What is the physical significance of the Jeans length? [5 marks]

(d) A density perturbation in an $\Omega = 1$ universe made of collisionless matter enters the horizon at the epoch of matter-radiation equality, $1 + z_{\rm eq} \simeq 1.3 \times 10^4$, with amplitude $\delta\rho/\rho = 10^{-6}$. Estimate the physical radius of the perturbation at horizon entry and its amplitude at the epoch of recombination, $1 + z_{\rm rec} \simeq 1000$. [5 marks]

(e) Consider a dark matter halo that collapses at redshift z_c and has mean interior density $\rho/\overline{\rho}_c = 178$, where $\overline{\rho}_c$ is the mean cosmic density at z_c. Calculate the mean overdensity of the halo at the present day and estimate its velocity dispersion, also at the present day, in terms of Ω, H and the halo's radius and collapse redshift. [5 marks]

2. The equation that describes the evolution of linear perturbations in a non-relativistic fluid in the expanding universe is

$$\ddot{\delta} + 2H\dot{\delta} = \frac{\nabla^2_{\underline{r}}(\delta P)}{\overline{\rho}} + \nabla^2_{\underline{r}}\phi$$

where

$$\nabla^2_{\underline{r}}\phi = 4\pi \mathrm{G}\left(\delta\rho_{\mathrm{tot}} + \frac{3\delta P}{c^2}\right)$$

Here, the dots denote time derivatives, $\delta \equiv \delta\rho/\overline{\rho}$ is the density perturbation, $\overline{\rho}$ the mean density, H the Hubble parameter, δP the perturbation in the pressure, ϕ the perturbation in the gravitational potential and $\delta\rho_{\mathrm{tot}}$ the perturbation in the *total* density.

(a) Show that for a *dust* universe, the perturbation equation can be written as:

$$\delta'' + \frac{2-q}{a}\,\delta' - \frac{3}{2}\frac{\Omega}{a^2}\,\delta = 0 \qquad (1)$$

where the prime denotes the partial derivative with respect to the expansion factor a, Ω is the matter density parameter and $q = -a\ddot{a}/\dot{a}^2$ is the deceleration parameter. [12 marks]

(b) Find the general solution of equation (1) for a flat universe made of collisionless matter ($q = 1/2$). [6 marks]

(c) Find the solution of equation (1) for a flat universe dominated by uniformly distributed dark energy ($q = -1$). [6 marks]

(d) According to measurements of the microwave background radiation, at the epoch of recombination temperature fluctuations in the baryon component of the universe had amplitude of order $\delta T/T \sim 10^{-5}$. Estimate the amplitude of the corresponding baryon density fluctuations today and compare that to the amplitude required for objects like galaxy clusters to collapse. What does this imply for the nature of the dark matter? [6 marks]

3. (a) Define the power spectrum of the density field in the expanding universe and explain what is meant by a Gaussian random field. [6 marks]

(b) The two-point correlation function of an isotropic density field is related to the power spectrum, $P(\underline{k}) \equiv \left|\delta_k(\underline{k})\right|^2$, by the following expression:

$$\xi(\underline{r}) = \frac{1}{(2\pi)^3}\int P(\underline{k})e^{-i\underline{k}\cdot\underline{r}}\,d^3k$$

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Perform the angular integral for an isotropic density field to find the constant A and the function B in

$$\xi(r) = A \int_0^\infty P(k) B k^2 dk$$

where $r = |\underline{r}|$. [6 marks]

(c) Assume that $P(k) \propto k^n$, where n is the spectral index, and that the rms density fluctuation on mass scale M behaves as

$$\sigma \propto \left(\frac{\delta M}{M}\right) \propto [k^3 P(k)]^{1/2} \propto M^{-\alpha}$$

Find α as a function of the spectral index n. What value of n gives mass-invariant fluctuations? [6 marks]

(d) For a power spectrum of the form $P(k) \propto k^n$, show that

$$\xi(r) \propto r^{-(3+n)}$$

[6 marks]

(e) Consider fluctuations with power spectrum $P(k) \propto k^n$ that enter the horizon at $a_{\rm enter}$ with constant amplitude in the matter-dominated era. (At $a_{\rm enter}$, $M \simeq M_{\rm horizon}$, where $M_{\rm horizon}$ denotes the horizon mass.) Assuming that after horizon entry the fluctuation amplitude grows in proportion to the expansion factor, calculate the value of the spectral index n at a subsequent epoch (such as recombination). [6 marks]