Galaxy Formation – Problem GF1.1

A. Using the Friedmann equation show that if the energy density remains constant in a flat universe, then the universe expands exponentially. List two periods in the evolution of our universe when this solution may be relevant.
B. Show that, if $\Omega_{tot} = \Omega_m + \Omega_r + \Omega_\Lambda = 1$, then the matter density parameter varies with time as

$$\Omega_m = \frac{\Omega_{m0}}{\Omega_{m0} + a^{-1}\Omega_{r0} + a^3\Omega_{\Lambda 0}}$$

Assuming that $\Omega_{m0} = 0.3$, $\Omega_{r0} = 5 \times 10^{-5}$ and $\Omega_{\Lambda 0} = 0.7$, plot a graph showing the values of the three density parameters as a function of a. Use a linear scale for the y-axis ranging from 0 to 1. You can use any method you wish to make the plot, including an automated plotting routine.

C. The Jeans mass is $M_J \simeq 5M_\odot (T/0.003\text{K})^{3/2}(\rho/\rho_0)^{-1/2}$ where $\rho_0 \simeq 3.6 \times 10^{-28}\text{kgm}^{-3}$ is the current density of baryons. Assume that $\rho = \rho_0 a^{-3}$

(a) After recombination, but before reionization, $T \simeq 3000(a/0.001)^{-2}\text{K}$. Find an expression for the variation of the Jeans mass with expansion factor.

(b) After reionization, $T \simeq 10^4\text{K}$. Find an expression for the variation of the Jeans mass with expansion factor.

(c) Sketch the variation of the Jeans mass with expansion factor, assuming a reionization redshift corresponding to $a = 0.1$. Use logarithmic scales on both axes.