Problem 1 - hand in Fri 27th Jan 14:15

1a) If A_d and B_d are components of first order covariant tensors, show that their linear sum forms $C_d = aA_d + bB_d$ is also a first order covariant tensor while the product $C_{de} = A_d B_e$ forms a second order covariant tensor.[2 marks]

b) What values are implied by the repeated sums $\delta^i{}_i$, $\delta^a{}_a$, $\delta^A{}_A$ and $\delta^\alpha{}_\alpha$?
[2 marks]

c) A contravariant 1st order tensor A^i has components (0,1,2). A covariant 1st order tensor B_j has components (3,4,5). Write out all the components of $A^i B_j$ and $A^i B_i$. [2 marks]

d) Any point on the surface of a hyperbolic parabolid surface has position $\vec{r} = (u+v)\mathbf{i} + (u-v)\mathbf{j} + 2uv\mathbf{k}$. Obtain expressions for the (tangent) basis vectors for this surface $\vec{e}_A$ and $\vec{e}_B$ and use these to derive the components of the covariant metric tensor $g_{AB} = \vec{e}_A.\vec{e}_B$. [4 marks]