a) geodesic paths are force free paths so are inertial frames where SR works. [1 mark]

Any 2 from the list: Radial free fall, circular orbits, elliptical precessing orbits, hyperbolic paths. [1 mark]

b) covariant derivative of the metric tensor

$$\partial g_{ab}/\partial x^c = \frac{\partial}{\partial x^c}(\vec{e_a}.\vec{e_b})$$

$$= \frac{\partial \vec{e_a}}{\partial x^c}.\vec{e_b} + \vec{e_a}.\frac{\partial \vec{e_b}}{\partial x^c} = \Gamma^d_{ac}\vec{e_d}.\vec{e_b} + \Gamma^d_{bc}\vec{e_a}.\vec{e_d} = \Gamma^d_{ac}g_{db} + \Gamma^d_{bc}g_{ad}$$

$$\frac{\partial g_{ab}}{\partial x^c} - \Gamma^d_{ac}g_{db} - \Gamma^d_{bc}g_{ad} = 0$$

[1 mark]

The LHS has the same form as the second order covariant derivative so $g_{ab;c} = 0$. [1 mark]

c) proper distance is measured by measuring the ends at the same time so

$$ds^2 = R^2(t)[\frac{dr^2}{(1-kr^2)}]$$

$$ds = R(t)\frac{dr}{(1-kr^2)^{1/2}}$$

put one at zero, the other at r_e and at time $t=0$ (close by)

$$d_{prop} = R(0)\int_0^{r_e} \frac{dr}{(1-kr^2)^{1/2}} \approx R(0)r_e$$

[1 mark]

$$v_{prop} = \frac{d}{dt}d_{prop} \approx \dot{R}(0)r_e = \frac{\dot{R}(0)}{R(0)}r_e R(0) = H(0)d_{prop}$$

[1 mark]

d) Parallel transport of a vector - change neither the magnitude or direction ie slide the vector from one point to another. In curved space the vector can swing around, following the curvature of the space [1 mark]

to do differentiation of vectors/tensors we need to compare the components at one point with them at a separate point at small distance away - so need to parallel transport the vector to compare it.[1 mark]

e) curvature=gravitation. describe the source of gravity by the stress energy tensor. This is second order with covariant derivative of zero. [1 mark]

Describe curvature by the reimman curvature tensor - 4th order! contract to second order ricci tensor - but this has non-zero divergance. So use ricci tensor and curvature scalar normalised metric to get a valid tensor equations. [1 mark]