$$\frac{D\lambda^{\alpha}}{D\tau} = \frac{d\lambda^{\alpha}}{d\tau} + \Gamma^{\alpha}_{\beta\gamma}\lambda^{\beta}\frac{dx^{\gamma}}{d\tau} = 0$$

the tangent vector is $\lambda^{\alpha} = dx^{\alpha}/d\tau$ [1 mark]

so its absolute derivative gives the geodesic condition

$$\frac{d^2x^{\alpha}}{d\tau^2} + \Gamma^{\alpha}_{\beta\gamma}\frac{dx^{\beta}}{d\tau}\frac{dx^{\gamma}}{d\tau} = 0$$

[1 mark]

$$U^{\alpha}_{;\beta}U^{\beta} = 0 = \Big(\frac{\partial U^{\alpha}}{\partial x^{\beta}} + \Gamma^{\alpha}{}_{\beta\gamma}U^{\gamma}\Big)U^{\beta}$$

[1 mark]

$$\frac{dx^{\beta}}{d\tau}\frac{\partial U^{\alpha}}{\partial x^{\beta}} + \Gamma^{\alpha}{}_{\beta\gamma}\frac{dx^{\gamma}}{d\tau}\frac{dx^{\beta}}{d\tau} = 0$$

[1 mark]

$$\frac{dU^{\alpha}}{d\tau} + \Gamma^{\alpha}{}_{\beta\gamma}\frac{dx^{\gamma}}{d\tau}\frac{dx^{\beta}}{d\tau} = 0$$

[1 mark]

$$\frac{d^2x^{\alpha}}{d\tau^2} + \Gamma^{\alpha}{}_{\beta\gamma}\frac{dx^{\gamma}}{d\tau}\frac{dx^{\beta}}{d\tau} = 0$$

which is the same as the geodesic condition [1 mark]

$N^{\alpha}{}_{,\alpha} = 0$ in SR, so in GR the derivative changes to covariant derivative giving $N^{\alpha}{}_{;\alpha} = 0$ [1 mark]

The stress energy tensor for dust $T^{\alpha\beta} = N^{\alpha}U^{\beta}$. So

$$T^{\alpha\beta}{}_{;\alpha} = 0 = N^{\alpha}{}_{;\alpha}U^{\beta} + N^{\alpha}U^{\beta}{}_{;\alpha}$$

[1 mark]

But from above $N^{\alpha}{}_{;\alpha} = 0$ so $N^{\alpha}U^{\beta}{}_{;\alpha} = 0$ [1 mark]

hence $\rho_0 U^{\alpha}U^{\beta}{}_{;\alpha} = 0$ i.e. $U^{\alpha}U^{\beta}{}_{;\alpha} = U^{\beta}U^{\alpha}{}_{;\beta} = 0$ which is the geodesic condition, so dust particles follow geodesic paths [1 mark]